Portland Cement Association

CONCRETE INFORMATION

DESIGN OF CIRCULAR DOMES

The domes considered in this article are surfaces of revolution. In spherical and conoidal domes, the surface is described by revolving an arc of a circle. The center of the circle may be on the axis of rotation (spherical dome) or outside the axis (conoidal dome). Both types may or may not have a symmetrical lantern opening through the top. The edge of the shell around its base is usually provided with an edge member cast integrally with the shell.

The loadings considered on these domes are (1) uniform load per square foot of dome surface, and (2) variable load equal to zero at top and increasing at a uniform rate toward the base. In either case, the load is constant along any circle of latitude or "hoop."

Another type of dome is also included which is produced by revolving an ellipse around its minor axis. The tangent to the ellipse at the end of its major axis is vertical at which point the thrust is vertical also. No edge member is required for ring tension when the ellipse is discontinued at the end of its major axis. Whatever hoop tension this dome may have is taken by bars provided in the shell itself.

The loadings on the elliptical dome are (1) uniform load per square foot of dome surface corresponding to a uniform shell thickness, and (2) uniform load per square foot of horizontal projection of dome surface which is intended to apply to snow load.

Shells are considered so thin that they cannot develop bending moment,* yet they are assumed to be so thick that there is no danger of buckling. Only shapes and loads which are symmetrical about axis of rotation are taken into account, and stresses due to wind pressure, volume change and support displacement are ignored. These assumptions are generally made in design of circular domes.

Theoretically speaking, the shells are supposed to carry neither concentrated nor unsymmetrical loading. A collar load uniformly distributed along the perimeter of a concentric circle is not considered "concentrated loading." Great care should be taken to avoid dangerous loading conditions when the formwork is being lowered and removed. The shell should be let down as easily and as uniformly as possible.

Formulas have been presented and derived on basis of the assumptions discussed. But these formulas apply only at points of domes which are removed some distance from the discontinuous edges. At the edges the results from the formulas may be indicative but they are not accurate. The edge member and the adjacent hoop of the shell must have very nearly the same strain when they are cast integrally. The significance of this fact is ignored in the derivations, and the forces computed from the formulas are therefore subject to certain modifications. For illustration, the edge member at the base is always in tension, but the hoop force computed in the adjacent circular strip of shell may be in compression, yet they must both have the same strain. The problem may be still more complex to solve if consideration is also extended to include casting schedule and prestressing of the edge member.

In almost any dome design, the question arises of stress allowed in compression. For a 3,000-lb. concrete, ACI Code 1963 allows 1,350 psi for bending, 750 psi for bearing, and 638 psi for columns. But for compression in thin shells, stresses are seldom more than 150-200 psi. There appears to be no theoretical reasoning behind these limiting figures, and codes state nothing about stresses in domes. They are based merely on trends in past and present practice.

Several reasons exist for keeping stresses fairly low in circular domes. As mentioned, forces near edge members are rather indeterminate, volume change and settlement are ignored, and concentrated loads may create critical stresses. To cover these and similar contingencies, it is well to keep stresses rather low. On the other hand, there is seldom any need for allowing high stresses. The shell must be thick enough to allow space and protection for two layers of reinforcement, so 3½ in. is about as thin as any shell can be made. For the average type and size of dome built in the past, allowable stresses did not have to be high to get a reasonably thin shell and an economical construction, but higher stresses will undoubtedly be allowed in the future as better concrete is used and more uncertainties are eliminated from the theory of design.

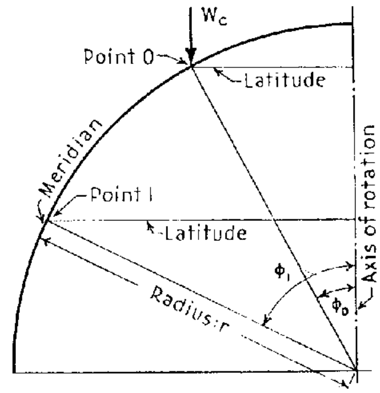
When the radius to thickness ratio exceeds 500, special consideration should be given in the design to the possible effects of buckling.

The data presented on design of domes are divided into three groups: formulas, numerical examples and derivations. A table is appended for use in design of elliptical domes. Beyond this, charts and tables have been avoided since the formulas are not long or cumbersome to handle.

*This discussion is based on membrane theory. For the bending theory, see textbooks on shells.

FORMULAS FOR SPHERICAL DOMES

FIG. 1



Consider part of dome between planes of latitude through Points 0 and 1 in Fig. 1.

Surface area: $A = 2\pi r^2 (\cos\phi_0 - \cos\phi_1)$ (1)

If load per sq.ft. of dome is uniform, w p.s.f.:

Total load: $W_u = 2\pi r^2 w (\cos\phi_0 - \cos\phi_1)$ (2)

If load increases from zero at Point 0 at a rate of w' per radian:

Total load: $W_v = 2\pi r^2 w' [\sin\phi_1 - \sin\phi_0 - \cos\phi_1 (\phi_1 - \phi_0)]$ (3)

Let W_c denote a collar load uniformly distributed around a circle of latitude. The general expression for total load, W, is then

$$W = W_u + W_v + W_c \quad (4)$$

If Point 0 lies on the axis of rotation:

$$A = 2\pi r^2 (1 - \cos\phi_1) \quad (1a)$$
$$W_u = 2\pi r^2 w (1 - \cos\phi_1) \quad (2a)$$
$$W_v = 2\pi r^2 w' (\sin\phi_1 - \phi_1 \cos\phi_1) \quad (3a)$$

Investigating forces at plane of latitude through Point 1, W being load above that circle:

$$\text{Meridional thrust: } T = \frac{W}{2\pi r \sin^2\phi_1} \quad (5)$$

$$\text{Hoop force: } H = -T + [w + w'(\phi_1 - \phi_0)]r\cos\phi_1 \quad (6)$$

If dome is discontinued along circle of latitude through Point 1, an edge member must be provided along that circle, and that member is subject to

$$\text{Ring tension: } S = \frac{W\cos\phi_1}{2\pi\sin\phi_1} \quad (7)$$

If dome is omitted above the circle of latititude through Point 0, the collar load W_c will produce a ring compression along the edge of the opening which equals

$$S = \frac{W_c \cos\phi_0}{2\pi\sin\phi_0} \quad (8)$$

The shell itself may be able to take this ring compression, but if an edge member is provided and cast integrally with the shell, it is customary to design it for the full amount of the force, S.

$$\text{At the top of a solid dome: } T = H = \frac{1}{2} wr \quad (9)$$

Example 1. Spherical Dome

Consider the spherical dome in Fig. 2. The rise is 25 ft., the radius of the base is 100 ft., and the live load is 30 p.s.f. Maximum allowable stress is 200 p.s.i.

Radius of dome surface: $r^2 = 100^2 + (r - 25)^2$

from which $r = 212.5$ ft.

Assume 5-in. uniform thickness of shell which gives:

$$w = 0.0625 + 0.030 = 0.0925 \text{ kip/sq.ft.}$$

From (9): $T_0 = H_0 = \frac{1}{2} \times 0.0925 \times 212.5 = 9.83$ kips/ft.

Forces at top of dome, Point 0.

$t_0 = h_0 \dfrac{9{,}830}{5 \times 12} = 164$ p.s.i. Stresses at Point 0.

At base, Point 1:

$\sin\phi_1 = \dfrac{100}{212.5} = 0.471$, $\sin^2\phi_1 = 0.222$, $\cos\phi_1 = 0.882$, $\phi_1 = 28$ deg.

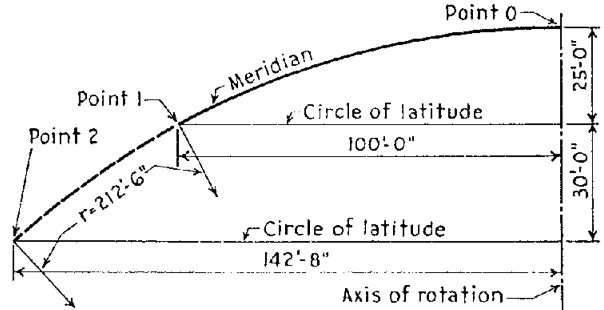


FIG. 2

From (2a): $W_{u1} = 2\pi \times 212.5^2 \times 0.0925 (1 - 0.882) = 3{,}100$ kips.

Total load above Point 1.

From (5):

$T_1 = \dfrac{3{,}100}{2\pi \times 212.5 \times 0.222} = 10.46$ kips/ft., $t_1 = \dfrac{10{,}460}{5 \times 12} = 175$ p.s.i.

Meridional stress at Point 1.

From (6):

$H_1 = -10.46 + 0.0925 \times 212.5 \times 0.882 = 6.88$ kips/ft. (compression)

Hoop force at Point 1.

From (7): $S_1 = \dfrac{3{,}100 \times 0.882}{2\pi \times 0.471} = 924$ kips.

Ring tension in edge member.

Assume that dome with $r = 212.5$ ft. is extended downward to where the rise is 55 ft. At the base at Point 2, the radius of the circumference is x: $212.5^2 = x^2 + (212.5 - 55)^2$. $x = 142.7$ ft.

$$\sin\phi_2 = \frac{142.7}{212.5} = 0.672,\ \sin^2\phi_2 = 0.452,\ \cos\phi_2 = 0.740$$

From (2a): $W_{u2} = 2\pi \times 212.5^2 \times 0.0925 (1 - 0.740) = 6{,}820$ kips.

Load at Point 2.

From (5):

$T_2 = \dfrac{6{,}820}{2\pi \times 212.5 \times 0.452} = 11.30$ kips/ft., $t_2 = \dfrac{11{,}300}{5 \times 12} = 188$ p.s.i.

Meridional stress at Point 2.

From (6):

$H_2 = -11.30 + 0.0925 \times 212.5 \times 0.740 = 3.25$ kips/ft. (compression)

Hoop force at Point 2.

From (7): $S_2 = \dfrac{6{,}820 \times 0.740}{2\pi \times 0.672} = 1{,}200$ kips.

Ring tension in edge member.

If the dome is one-half of a sphere, $\sin\phi_3 = 1.000$ and $\cos\phi_3 = 0.000$ which gives:

$W_{u3} = 26{,}240$ kips, $T_3 = 19.65$ kips/ft., $t_3 = 328$ p.s.i., $H_3 = -19.65$ kips/ft. (tension), $S_3 = 0$ (no edge member needed for ring tension)

When $H = 0$: $T = wr\cos\phi$, or $\dfrac{2\pi r^2 w(1 - \cos\phi)}{2\pi r \sin^2\phi} = wr\cos\phi$

from which $\cos\phi = \dfrac{\sqrt{5} - 1}{2}$, or $\phi = 52$ deg.

Example 2. Spherical Dome

The data are the same as in Example 1 except that the dome has a lantern opening arranged symmetrically around the axis of rotation. The radius of the opening is 25 ft. A collar load is arranged uniformly around the edge of opening, and the total collar load is 50 kips.

Instead of Point 0 being on the axis of rotation as in Example 1, it is here lying on the edge of the opening and

$$\sin\phi_0 = \frac{25.0}{212.5} = 0.1176,\ \sin^2\phi_0 = 0.0138,\ \cos\phi_0 = 0.993$$

Point 1 is considered in the same position as in Example 1, so

$$\sin\phi_1 = 0.471,\ \sin^2\phi_1 = 0.222,\ \cos\phi_1 = 0.882$$

From (4) and (2):

$W_1 = 2\pi \times 212.5^2 \times 0.0925 (0.993 - 0.882) + 50 = 2{,}960$ kips.

Total load above Point 1.

T_1, H_1 and S_1 may be computed as in Example 1 substituting 2,960 for 3,100 kips.

From (8): $S_0 = \dfrac{50 \times 0.993}{2\pi \times 0.1176} = 67$ kips.

Compression in edge member at lantern opening.

Example 3. Spherical Dome

The data are the same as in Example 1 except that the dome thickness varies from 5 in. at top (Point 0) to 6 in. at base (Point 1) with radius of 100 ft. Determine forces and stresses at base.

A 5-in. thickness considered uniform throughout plus a 30-lb. live load amounts to 0.0925 kip/sq.ft. In addition, the weight of the slab increases from zero at Point 0 ($\phi_0 = 0$) to 0.0125 kip/sq.ft. at Point 1. ($\phi_1 = 0.490$ radian*). The rate of increase per radian is $w' = 0.0125 \div 0.490 = 0.0255$ kip/sq.ft.

From (2), (3), (4): $W_1 = 2\pi \times 212.5^2 \times 0.0925\ (1 - 0.882) + 2\pi \times 212.5^2 \times 0.0255\ (0.471 - 0.490 \times 0.882)$
$= 3{,}380$ kips Total load above Point 1.

From (5): $T_1 = \dfrac{3{,}380}{2\pi \times 212.5 \times 0.222} = 11.40$ kips/sq.ft.;

$t_1 = \dfrac{11{,}400}{6 \times 12} = 158$ p.s.i. Meridional stress at Point 1.

From (6): $H_1 = -11.40 + (0.0925 + 0.0255 \times 0.490)\ 212.5 \times 0.882$
$= 8.28$ kips/ft. (compression). Hoop force at Point 1.

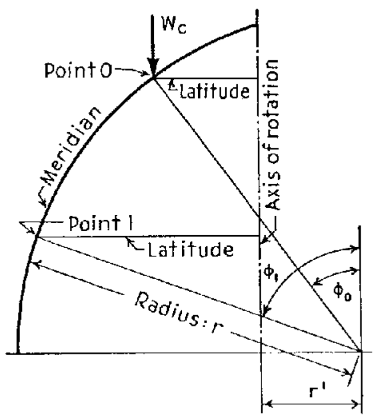
From (7): $S_1 = \dfrac{3{,}380 \times 0.882}{2\pi \times 0.471} = 1{,}007$ kips
Ring tension in edge member.

Assume the dome has a lantern opening as in Example 2 and Point 0 lies on the edge of the opening at which the shell is 5 in. thick. Compute forces at base, Point 1.

$\phi_0 = 0.118$, $w' = 0.0125 \div (0.490 - 0.118) = 0.0336$ kips/sq.ft.
$W_1 = 2\pi \times 212.5^2 \times 0.0925\ (0.993 - 0.882)$
$+ 2\pi \times 212.5^2 \times 0.0336\ (0.471 - 0.118 - 0.882 \times 0.372)$
$= 3{,}150$ kips.
(Add hereto the collar load, W_c. In this case, set $W_c = 0$).
$T_1 = 10.63$ kips/ft., $H_1 = 9.05$ kips/ft., $S_1 = 939$ kips.

FORMULAS FOR CONOIDAL DOMES

FIG. 3



Consider part of dome between planes of latitude through Points 0 and 1 in Fig. 3.

Surface area: $A = 2\pi r^2 (\cos\phi_0 - \cos\phi_1) - 2\pi r r' (\phi_1 - \phi_0)$ (10)

If load per sq.ft. of dome is uniform, w p.s.f.:

Total load: $W_u = 2\pi r^2 w (\cos\phi_0 - \cos\phi_1) - 2\pi r r' w (\phi_1 - \phi_0)$ (11)

If load increases from zero at Point 0 at a rate of w' per radian:

Total load:

$W_v = 2\pi r^2 w' [\sin\phi_1 - \sin\phi_0 - \cos\phi_1 (\phi_1 - \phi_0)] - \pi r r' w' (\phi_1 - \phi_0)^2$ (12)

Let W_c denote a collar load uniformly distributed around a circle of latitude. The general expression for total load, W, is then

$$W = W_u + W_v + W_c \quad (13)$$

If Point 0 lies on the axis of rotation, ϕ_0 is determined by the equation

$$\sin\phi_0 = \frac{r'}{r} \quad (14)$$

Investigating forces at circle of latitude through Point 1, W being load above that circle:

$$\text{Meridional thrust: } T = \frac{W}{2\pi\ (r \sin\phi_1 - r')\sin\phi_1} \quad (15)$$

*One radian equals 57.3 deg.

Hoop force:

$$H = \frac{r\sin\phi_1 - r'}{r\sin\phi_1}\left[-T + [w + w'(\phi_1 - \phi_0)]\ r\cos\phi_1\right] \quad (16)$$

If dome is discontinued along circle of latitude through Point 1, an edge member must be provided along that circle, and that member is subject to

$$\text{Ring tension: } S = \frac{W\cos\phi_1}{2\pi\sin\phi_1} \quad (7)$$

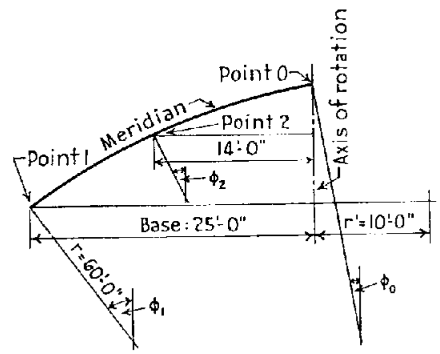
If dome is omitted above the circle of latitude through Point 0, the collar load W_c will produce a ring compression, S, along the edge of the opening (*see* Spherical Domes *for additional notes*):

$$\text{Ring compression: } S = \frac{W_c\cos\phi_0}{2\pi\sin\phi_0} \quad (8)$$

At top of dome where it intersects the axis of rotation, the formulas given above do not apply because there is local bending in the vicinity of the peak. There may also be some local bending near an edge member.

Example 4. Conoidal Dome

FIG. 4



Consider the conoidal dome in Fig. 4. Radius of base of dome, or distance from Point 1 to axis of rotation, is 25 ft. Radius of dome is $r = 60$ ft., and distance from center of dome surface to axis of rotation is $r' = 10$ ft. Thickness of dome is 3½ in. Live load is 30 p.s.f. Point 0 lies on axis of rotation, so $\sin\phi_0 = r' \div r = 10 \div 60 = 0.167$, $\cos\phi_0 = 0.986$, and $\phi_0 = 0.167$ radian.

$\sin\phi_1 = (25.0 + 10.0) \div 60.0 = 0.583$, $\cos\phi_1 = 0.812$,
$\phi_1 = 0.623$ radian. $w = 0.044 + 0.030 = 0.074$ kip/sq.ft.

From (11): $W_{u1} = 2\pi \times 60.0^2 \times 0.074\ (0.986 - 0.812)$
$- 2\pi \times 60.0 \times 10.0 \times 0.074\ (0.623 - 0.167)$
$= 291 - 127 = 164$ kips. Total load above Point 1.

From (15): $T_1 = \dfrac{164}{2\pi\ (60.0 \times 0.583 - 10.0) \times 0.583} = 1.79$ kips/ft.
Meridional thrust at Point 1.

From (16):
$H_1 = \dfrac{60.0 \times 0.583 - 10.0}{60.0 \times 0.583}\ (-1.79 + 0.074 \times 60.0 \times 0.812)$
$= +1.30$ kips/ft. Hoop compression at Point 1.

From (7): $S_1 = \dfrac{164 \times 0.812}{2\pi \times 0.583} = 36.4$ kips
Ring tension in edge member.

Compute forces at a Point 2 at which:
$\sin\phi_2 = 0.400$, $\cos\phi_2 = 0.917$, $\phi_2 = 0.412$ radian.

From (11): $W_{u2} = 2\pi \times 60.0^2 \times 0.074\ (0.986 - 0.917)$
$- 2\pi \times 60.0 \times 10.0 \times 0.074\ (0.412 - 0.167)$
$= 115 - 68 = 47$ kips. Total load above Point 2.

From (15): $T_2 = \dfrac{47}{2\pi \times (60.0 \times 0.400 - 10.0)\ 0.400} = 1.34$ kips/ft.
Meridional thrust at Point 2.

From (16):
$H_2 = \dfrac{60.0 \times 0.400 - 10.0}{60.0 \times 0.400}\ (-1.34 + 0.074 \times 60.0 \times 0.917)$
$= +1.59$ kips/ft. Hoop compression at Point 2.

Example 5. Conoidal Dome

The data are the same as in Example 4 except that the dome has a lantern opening the edge of which follows a circle of latitude with radius of 7 ft. Point 0 lies on this circle, so $\sin\phi_0 = (7 + 10) \div 60 = 0.283$, $\cos\phi_0 = 0.959$, $\phi_0 = 0.287$. Also, there is a collar load of 0.4 kip/ft. along the edge of the opening. Only part of the calculations will be illustrated.

At Point 1: $\sin\phi_1 = 0.583$, $\cos\phi_1 = 0.812$, $\phi_1 = 0.623$ radian.
$w = 0.074$ kip/sq.ft.

From (11), (13):

$$W_1 = 2\pi \times 60.0^2 \times 0.074\,(0.959 - 0.812) - 2\pi \times 60.0 \times 10.0 \times 0.074\,(0.623 - 0.287) + 0.4 \times 2\pi \times 7.0$$
$$= 246 - 94 + 18 = 170 \text{ kips.}$$

Total load above Point 1.

T_1 and S_1 may be computed as in Example 4 substituting 170 for 164 kips.

From (8): $S_0 = \dfrac{18 \times 0.959}{2\pi \times 0.283} = 10$ kips.

Compression in edge member at opening.

Example 6. Conoidal Dome

The data are the same as in Example 4 except that the dome thickness varies from 3 in. at top (Point 0: $\sin\phi_0 = 0.167$, $\cos\phi_0 = 0.986$, $\phi_0 = 0.167$) to 4 in. at base (Point 1: $\sin\phi_1 = 0.583$, $\cos\phi_1 = 0.812$, $\phi_1 = 0.623$). The uniform load is $w = 0.0375 + 0.030 = 0.0675$ kip/sq.ft. In addition, the slab dead load increases from 0 at top to 0.0125 at base. The rate of increase per radian is

$$w' = \frac{0.0125}{0.623 - 0.167} = 0.0274 \text{ kip/sq.ft.}$$

From (11), (12):

$$W_1 = 2\pi \times 60.0^2 \times 0.0675\,(0.986 - 0.812) - 2\pi \times 60.0 \times 10.0 \times 0.0675\,(0.623 - 0.167)$$
$$+ 2\pi \times 60.0^2 \times 0.0274\,[0.583 - 0.167 - 0.812\,(0.623 - 0.167)] - \pi \times 60.0 \times 10.0 \times 0.0274\,(0.623 - 0.167)^2$$
$$= 265.7 - 116.0 + 28.5 - 10.7$$
$$= 167.5, \text{ say } 168 \text{ kips.}$$

Total load above Point 1.

T_1 and S_1 at base may be computed from (15) and (7) inserting $W = 168$ kips.

Investigate forces at Point 2: $\sin\phi_2 = 0.400$, $\cos\phi_2 = 0.917$, $\phi_2 = 0.412$ radian.

From (11), (12):

$$W_1 = 2\pi \times 60.0^2 \times 0.0675\,(0.986 - 0.917) - 2\pi \times 60.0 \times 10.0 \times 0.0675\,(0.412 - 0.167)$$
$$+ 2\pi \times 60.0^2 \times 0.0274\,[0.400 - 0.167 - 0.917\,(0.412 - 0.167)] - \pi \times 60.0 \times 10.0 \times 0.0274\,(0.412 - 0.167)^2$$
$$= 105.4 - 62.3 + 5.1 - 3.1$$
$$= 45.1, \text{ say, } 45 \text{ kips.}$$

Total load above Point 2.

From (15): $T_2 = \dfrac{45}{2\pi\,(60.0 \times 0.400 - 10.0)\,0.400}$
$= 1.28$ kips/ft. Meridional thrust at Point 2.

From (16): $$H_2 = \frac{60.0 \times 0.400 - 10.0}{60.0 \times 0.400}\Big[-1.28 + [0.0675 + 0.0274\,(0.412 - 0.167)]\,60.0 \times 0.916\Big]$$
$= +1.63$ kips/ft. Hoop compression at Point 2.

FORMULAS FOR ELLIPTICAL DOMES

Consider part of dome above circle of latitude through Point 1 in Fig. 5.
Surface area: $A = 2\pi a^2 C$.

Select values of C (and Q) from table based on computed ratios of $\dfrac{b}{a}$ and $g = \dfrac{y}{b}$

Let w first be uniform weight per sq.ft. of dome surface.

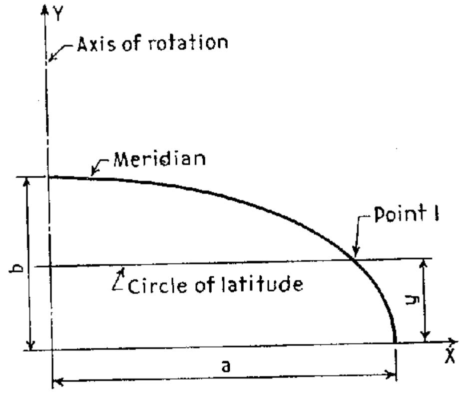
Total weight above Point 1: $W = 2\pi a^2 w C$ (17)

Meridional thrust at Point 1: $T = \dfrac{wa^2}{b} \times \dfrac{CQ}{1 - g^2}$ (18)

Hoop force at Point 1: $H = \dfrac{wa^2}{b}\left(g - \dfrac{C}{(1 - g^2)\,Q}\right)$ (19)

At axis of rotation: $T = H = \dfrac{wa^2}{2b}$ (20)

FIG. 5



If a symmetrical lantern opening with radius x is cut through the shell at the crown, proceed as follows: From W, computed in accordance with (17), deduct $\pi w x^2$ and add collar load if any. Multiply T computed from (18) by the ratio of W'/W which gives T'. Determine hoop force from

$$H' = \frac{wa^2}{b}\,g - \frac{T'}{Q^2}$$

Note: W' is total weight above point considered regardless of the source from which it is derived.

Now let w denote weight per sq.ft. uniformly distributed over horizontal projection of dome.

Total weight above Point 1: $W = \pi a^2 w\,(1 - g^2)$ (21)

Meridional thrust at Point 1: $T = \dfrac{wa^2}{2b}Q$ (22)

Hoop force at Point 1: $H = \dfrac{wa^2}{2b} \times \dfrac{2g^2 - 1}{Q}$ (23)

At axis of rotation: $T = H = \dfrac{wa^2}{2b}$ (24)

If dome is discontinued along circle of latitude through Point 1, an edge member must be provided along that circle, and that member is subject to

Ring tension: $S = \dfrac{Wa}{2\pi b} \times \dfrac{g}{\sqrt{1 - g^2}}$ (25)

If dome is omitted above a circle of latitude and a collar load W_c is applied uniformly along that circle, W_c will produce a ring compression along the edge of the opening which equals

$$S = \frac{W_c a}{2\pi b} \times \frac{g}{\sqrt{1 - g^2}}$$

Example 7. Elliptical Dome

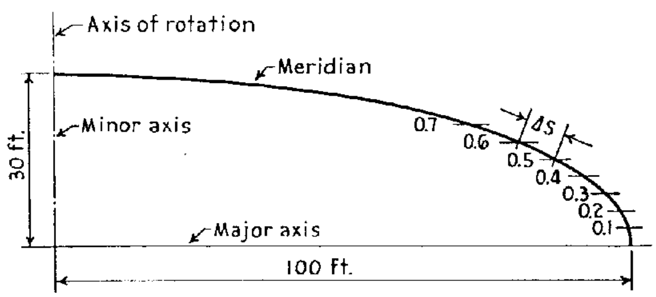


FIG. 6

Consider the elliptical dome in Fig. 6. The live load (LL) is 30 p.s.f. of horizontal projection. The shell is 5 in. thick, weighing 62.5 p.s.f. ($= DL$)

Forces at end of minor axis:

From (20) for DL and (24) for LL:

$$T = H = \frac{0.0625 \times 100^2}{2 \times 30} + \frac{0.030 \times 100^2}{2 \times 30} = 15.4 \text{ kips/ft.}$$

Table of Coefficients for Elliptical Domes

$g = \frac{y}{b}$	Values of C* $\frac{b}{a}$ = 0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	Values of Q** $\frac{b}{a}$ = 0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	$g = \frac{y}{b}$
0.0	.547	.588	.636	.690	.747	.807	.870	.934	1.000	.200	.300	.400	.500	.600	.700	.800	.900	1.000	0.0
0.1	.526	.558	.596	.640	.687	.737	.789	.844	.900	.227	.315	.410	.507	.605	.704	.802	.901	1.000	0.1
0.2	.501	.525	.554	.588	.626	.666	.709	.754	.800	.280	.356	.440	.529	.621	.714	.809	.904	1.000	0.2
0.3	.469	.486	.508	.534	.563	.594	.627	.663	.700	.356	.415	.485	.563	.646	.732	.820	.909	1.000	0.3
0.4	.430	.441	.456	.475	.496	.520	.545	.572	.600	.440	.485	.543	.608	.680	.756	.835	.917	1.000	0.4
0.5	.381	.389	.399	.412	.427	.443	.461	.480	.500	.529	.563	.608	.661	.721	.786	.854	.926	1.000	0.5
0.6	.323	.328	.335	.343	.352	.362	.374	.387	.400	.621	.646	.680	.721	.768	.821	.877	.937	1.000	0.6
0.7	.257	.260	.263	.267	.272	.278	.285	.292	.300	.714	.732	.756	.786	.821	.860	.904	.950	1.000	0.7
0.8	.181	.182	.183	.185	.188	.190	.193	.196	.200	.809	.820	.835	.854	.877	.903	.933	.965	1.000	0.8
0.9	.095	.095	.096	.096	.097	.098	.098	.099	.100	.904	.910	.917	.926	.937	.950	.965	.982	1.000	0.9
1.0	.000	.000	.000	.000	.000	.000	.000	.000	.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.0

$$^{*}C = \frac{1}{2} + \frac{1-k^2}{2k}\log(1+k) - \frac{g}{2}\sqrt{1-k^2(1-g^2)} - \frac{1-k^2}{2k}\log\left(gk + \sqrt{1-k^2(1-g^2)}\right), \text{ in which } k^2 = 1 - \frac{b^2}{a^2}$$

$$^{**}Q = \sqrt{1-k^2(1-g^2)}$$

Stresses: $t = h = \dfrac{15{,}400}{5 \times 12} = 257$ p.s.i.

Forces at end of major axis: Select coefficients from table for values of $\frac{b}{a} = 30 \div 100 = 0.3$ and $g = \frac{y}{b} = 0$. All calculations have two terms, dead load first and live load second. From chart: $C = 0.588$ and $Q = 0.300$.

From (17) and (21):

$$W = 2\pi 100^2 \times 0.0625 \times 0.588 + \pi 100^2 \times 0.030 \times (1-0) = 2{,}310 + 940 = 3{,}250 \text{ kips.}$$

From (18) and (22):

$$T = \frac{0.0625 \times 100^2}{30} \times \frac{0.588 \times 0.300}{1-0} + \frac{0.030 \times 100^2}{2 \times 30} \times 0.300 = 3.68 + 1.50 = 5.18 \text{ kips/ft.}$$

From (19) and (23):

$$H = \frac{0.0625 \times 100^2}{30}\left(0 - \frac{0.588}{(1-0) \times 0.300}\right) + \frac{0.030 \times 100^2}{2 \times 30} \times \frac{0-1}{0.300} = -40.8 - 16.7 = -57.5 \text{ kips/ft.}$$

Example 8. Elliptical Dome

Same data as in Example 7. Compute hoop tension forces. Divide the 30-ft. rise into ten equal parts. At the circles of latitude corresponding to values of g from 0.0 to 0.7 (*see table above*) compute:

From (19) for dead load ($w = 0.0625$): $H = \dfrac{wa^2}{b}\left(g - \dfrac{C}{(1-g^2)Q}\right)$

From (23) for live load ($w = 0.030$): $H = \dfrac{wa^2}{2b} \times \dfrac{2g^2-1}{Q}$

These computations are tabulated at right. In addition, it is necessary to determine the length between the intersection points measured along the meridian. Calling each length Δs as indicated in Fig. 6, we have

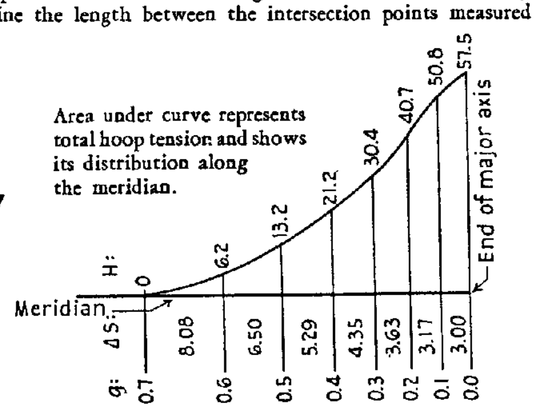
$$\Delta s = \frac{\Delta y}{\sin\theta} = \frac{aQ}{b\sqrt{1-g^2}} \times \Delta y \text{ (in which } a = 100,\ b = 30,\ \Delta y = 3,$$

therefore, $\Delta s = \dfrac{10Q}{\sqrt{1-g^2}}$

Values of Δs are also tabulated and are used together with H-values to plot the hoop tension diagram in Fig 7.

$g = \frac{y}{b}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
C (*see table*)	0.588	0.558	0.525	0.486	0.441	0.389	0.328	0.260
Q (*see table*)	0.300	0.315	0.356	0.415	0.485	0.563	0.646	0.732
$w\left[g - \frac{C}{(1-g^2)Q}\right]$	−.1225	−.1056	−.0834	−.0617	−.0426	−.0263	−.0121	+.0003
$w\frac{2g^2-1}{2Q}$	−.0500	−.0467	−.0388	−.0296	−.0210	−.0133	−.0065	−.0004
Summation	−.1725	−.1523	−.1222	−.0913	−.0636	−.0396	−.0186	−.0001
$\frac{a^2}{b} \times$ Sum $= H$	57.5	50.8	40.7	30.4	21.2	13.2	6.2	0.0
$\Delta s = \frac{10Q}{\sqrt{1-g^2}}$	3.00	3.17	3.63	4.35	5.29	6.50	8.08	—

FIG. 7



DERIVATIONS FOR SPHERICAL DOMES

0-1: arc of circle with radius r (*see Fig. 8.*)

Center of circle lies on axis of rotation.

ϕ: angle measured from axis of rotation.

$d\phi$: small increment of angle.

$rd\phi$: length of element of arc.

$r\sin\phi$: distance from element to axis of rotation.

$rd\phi \times 2\pi r \sin\phi$: area of element rotated about axis.

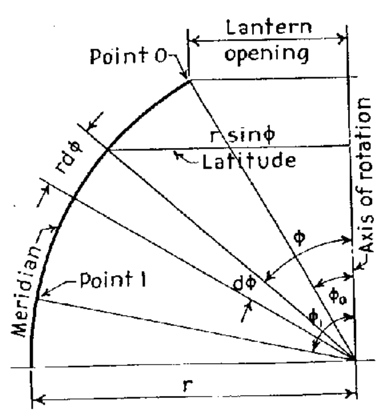


FIG. 8

Total surface area of spherical dome described by rotating Arc 0-1 about axis:

$$A = \int_{\phi_0}^{\phi_1} r\,d\phi \times 2\pi r \sin\phi = 2\pi r^2 \int_{\phi_0}^{\phi_1} \sin\phi\,d\phi = 2\pi r^2 \Big[-\cos\phi\Big]_{\phi_0}^{\phi_1}$$

$$= 2\pi r^2 (\cos\phi_0 - \cos\phi_1) \qquad (1)$$

If Point 0 lies on axis of rotation, $\cos\phi_0 = 1$, and

$$A = 2\pi r^2 (1 - \cos\phi_1) \qquad (1a)$$

If the unit load, w, is the same for all elements, the total load, W_u on the dome between Points 0 and 1 is

$$W_u = w \times A = 2\pi r^2 \times w(\cos\phi_0 - \cos\phi_1) \qquad (2)$$

If the unit load increases from zero at Point 0 at a uniform rate of w' per radian, the unit load at the element $r d\phi$ equals $w'(\phi - \phi_0)$.
The load on the element of dome described by rotating $r d\phi$ about the axis equals

$$r d\phi \times 2\pi r \sin\phi \times w'(\phi - \phi_0)$$

Total load of spherical dome described by rotating Arc 0-1 about axis:

$$W_v = \int_{\phi_0}^{\phi_1} r d\phi \times 2\pi r \sin\phi \times w'(\phi - \phi_0)$$

$$= 2\pi r^2 \times w' \int_{\phi_0}^{\phi_1} \phi \sin\phi\,d\phi - 2\pi r^2 \times w' \times \phi_0 \int_{\phi_0}^{\phi_1} \sin\phi\,d\phi$$

$$W_v = 2\pi r^2 \times w' \Big[-\phi\cos\phi + \sin\phi\Big]_{\phi_0}^{\phi_1} - 2\pi r^2 \times w'\phi_0 \Big[-\cos\phi\Big]_{\phi_0}^{\phi_1}$$

$$W_v = 2\pi r^2 \times w' \times$$
$$(-\phi_1\cos\phi_1 + \sin\phi_1 + \phi_0\cos\phi_0 - \sin\phi_0 + \phi_0\cos\phi_1 - \phi_0\cos\phi_0)$$

$$W_v = 2\pi r^2 \times w' \,[\sin\phi_1 - \sin\phi_0 - \cos\phi_1\,(\phi_1 - \phi_0)] \qquad (3)$$

If Point 0 lies on axis of rotation, $\phi_0 = 0$, $\cos\phi_0 = 1$, and

$$W_v = 2\pi r^2 \times w' \,(\sin\phi_1 - \phi_1\cos\phi_1) \qquad (3a)$$

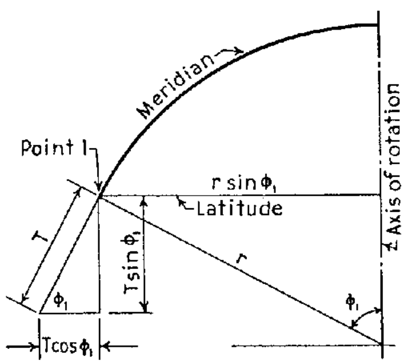
Meridional Thrust and Hoop Forces

W: load above plane of latitude through Point 1 (*see Fig. 9*).
T: meridional thrust per unit length of circle of latitude through Point 1.
$2\pi r \sin\phi_1$: length of circle of latitude through Point 1.
$T\sin\phi_1$: vertical component of T.
Set W equal to sum of vertical components of T:

$$W = 2\pi r \sin\phi_1 \times T\sin\phi_1 = 2\pi r \sin^2\phi_1 \times T.$$

$$T = \frac{W}{2\pi r \times \sin^2\phi_1} \qquad (5)$$

FIG. 9



If the dome is discontinued along circle of latitude through Point 1, a circular ring through that point is subject to a unit radial force of $T\cos\phi_1$.
A unit radial pressure, s, on a circular ring with radius, R, causes a ring tension of $S = s \times R$.
Set $s = T\cos\phi_1$ and $R = r\sin\phi_1$ which gives the ring tension in the edge member at Point 1:

$$S = T\cos\phi_1 \times r\sin\phi_1 = \frac{W \times \cos\phi_1}{2\pi r \sin^2\phi_1} \times r\sin\phi_1 = \frac{W\cos\phi_1}{2\pi\sin\phi_1} \qquad (7)$$

At Point 0, on the axis of rotation, consider an elemental square with weight w subject to a thrust T on each of its four sides. Insert W_u from (2) in (5), and set $\sin^2\phi = (1 - \cos\phi)(1 + \cos\phi)$:

$$T = \frac{2\pi r^2 \times w\,(1 - \cos\phi)}{2\pi r\,(1 - \cos\phi)\,(1 + \cos\phi)} = \frac{wr}{1 + \cos\phi} = \frac{1}{2}wr \qquad (9)$$

An elemental square at Point 1 is subject to three forces:

(1) meridional thrust, T, tangential to meridian.
(2) hoop force, H, tangential to circle of latitude.
(3) load on element, $w + w'(\phi_1 - \phi_0)$.

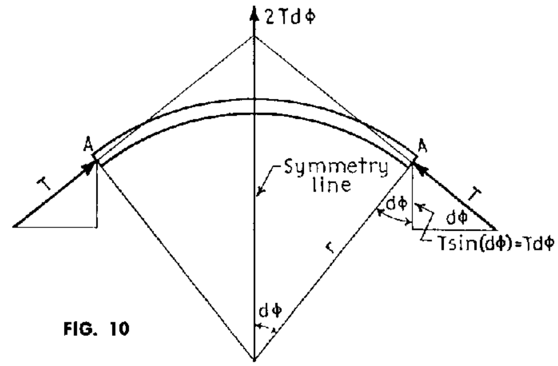


FIG. 10

Fig. 10 shows an arc A - A which is part of a circular ring with radius r. The compression in the ring is T, and the angle subtending the arc is $2d\phi$, $d\phi$ being considered such a small angle that $\sin(d\phi) = d\phi$. The present object is to determine the radial component of the T-forces. It is seen from the triangles in the figure that the component of T in the direction parallel to the symmetry line is $T\sin(d\phi)$ which equals $Td\phi$ since $d\phi$ is a small angle. The total radial component is $2Td\phi$ on an arc A - A of length $2rd\phi$. The radial component for an arc of unit length therefore is

$$\frac{2Td\phi}{2rd\phi} = \frac{T}{r}$$

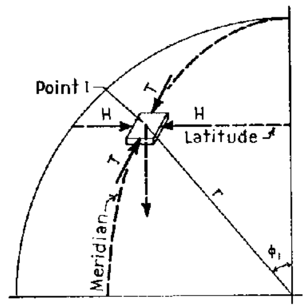


FIG. 11

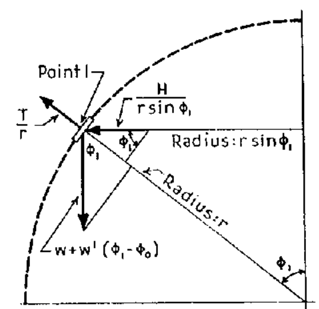


FIG. 12

In regard to equilibrium of an element of the dome, the tangential force may be replaced by its radial component $\frac{T}{r}$ (*Figs. 11 and 12*). The H-force is tangential to the circle of latitude with radius $r\sin\phi_1$ and may be replaced by its radial component lying in the plane of the circle and equaling $\frac{H}{r\sin\phi_1}$. The two components and the load on the element lie in the same vertical plane. Since the element is in equilibrium the sum of projections of the three forces must equal zero. Projecting on the line through center of the dome gives

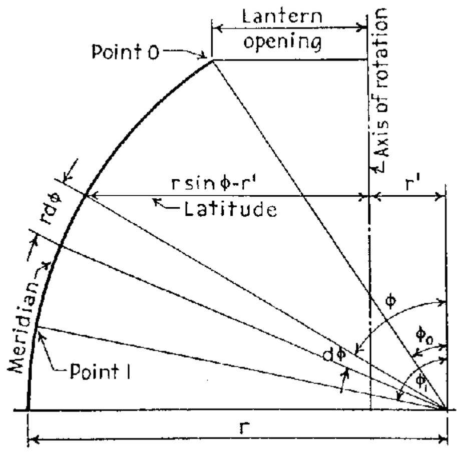
$$\frac{T}{r} + \frac{H}{r\sin\phi_1} \times \sin\phi_1 - [w + w'(\phi_1 - \phi_0)]\cos\phi_1 = 0$$

$$\text{or } H = -T + [w + w'(\phi_1 - \phi_0)]\,r\cos\phi_1 \qquad (6)$$

DERIVATIONS FOR CONOIDAL DOMES

The derivations follow the same general procedure as illustrated for spherical domes. The principal difference is in the location of the center for the Arc 0 - 1. The distance from the center to the axis of rotation is called r' (*see Fig. 13*).

FIG. 13



When $r' = 0$, the dome is spherical; when $r \neq 0$, it is conoidal. The radius of the circle of latitude is $r \sin\phi - r'$ for conoidal domes. Substitute this quantity for $r \sin\phi$ used in spherical domes:

$$A = \int_{\phi_0}^{\phi_1} r d\phi \times 2\pi\,(r\sin\phi - r') = 2\pi r^2 \int_{\phi_0}^{\phi_1} \sin\phi\, d\phi - 2\pi r r' \int_{\phi_0}^{\phi_1} d\phi$$

$$A = 2\pi r^2 (\cos\phi_0 - \cos\phi_1) - 2\pi r r'\,(\phi_1 - \phi_0) \qquad (10)$$

If load is uniform per sq.ft. of dome, w, total load of dome between Points 0 and 1 is

$$W_u = 2\pi r^2 \times w\,(\cos\phi_0 - \cos\phi_1) - 2\pi r r' \times w\,(\phi_1 - \phi_0) \qquad (11)$$

If the unit load increases from zero at Point 0 at a uniform rate of w' per radian, the total load on a conoidal dome between Points 0 and 1 is:

$$W_v = \int_{\phi_0}^{\phi_1} r d\phi \times 2\pi\,(r\sin\phi - r') \times w'\,(\phi - \phi_0)$$

$$W_v = 2\pi r^2 \times w' \int_{\phi_0}^{\phi_1} \phi \sin\phi\, d\phi - 2\pi r^2 \times w'\phi_0 \int_{\phi_0}^{\phi_1} \sin\phi\, d\phi$$
$$- 2\pi r \times w'r' \int_{\phi_0}^{\phi_1} \phi\, d\phi + 2\pi r \times w'\,\phi_0 r' \int_{\phi_0}^{\phi_1} d\phi$$

The first two terms are the same as those integrated for spherical domes and are equal to

$$2\pi r^2 \times w'\,[\sin\phi_1 - \sin\phi_0 - \cos\phi_1\,(\phi_1 - \phi_0)]$$

The last two terms equal:

$$- 2\pi r \times w'r'\left(\frac{\phi_1^2}{2} - \frac{\phi_0^2}{2}\right) + 2\pi r \times w'r' \times \phi_0\,(\phi_1 - \phi_0)$$
$$= -\pi r r' \times w'\,(\phi_1 - \phi_0)^2$$

$$W_v = 2\pi r^2 \times w'\,[\sin\phi_1 - \sin\phi_0 - \cos\phi_1\,(\phi_1 - \phi_0)] - \pi r r' \times w'\,(\phi_1 - \phi_0)^2 \qquad (12)$$

For derivation of equation for meridional thrust, T, refer to spherical domes but substitute $r\sin\phi_1 - r'$ for $r\sin\phi_1$ as follows:

$$W = 2\pi\,(r\sin\phi_1 - r')\,T\sin\phi_1$$

$$T = \frac{W}{2\pi\,(r\sin\phi_1 - r')\sin\phi_1} \qquad (15)$$

The hoop force, H, may be derived as for spherical domes:

$$\frac{T}{r} + \frac{H}{r\sin\phi_1 - r'} \times \sin\phi_1 - [w + w'\,(\phi_1 - \phi_0)]\cos\phi_1 = 0$$

$$H = \frac{r\sin\phi_1 - r'}{r\sin\phi_1}\left[-T + [w + w'\,(\phi_1 - \phi_0)]\,r\cos\phi_1\right] \qquad (16)$$

The formulas for forces in edge members are the same as for spherical domes.

DERIVATIONS FOR ELLIPTICAL DOMES

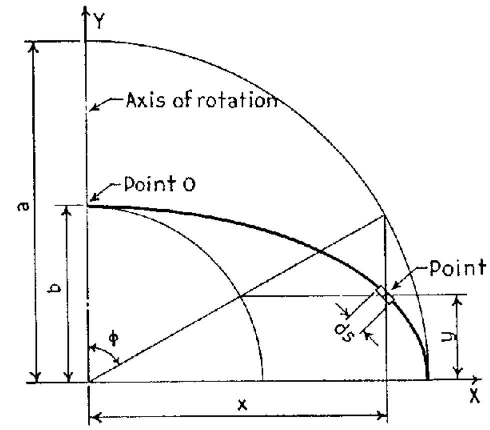
Equation of ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in which

$x = a\sin\phi \qquad dx = a\cos\phi\, d\phi$

$y = b\cos\phi \qquad dy = -\, b\sin\phi\, d\phi$

$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{a^2\cos^2\phi + b^2\sin^2\phi} \times d\phi$

FIG. 14



Consider ring produced by rotating element ds about Y-axis. Weight per sq.ft. of dome surface is w, uniformly distributed. Weight of elemental ring is $dw = 2\pi x w ds$ (*see Fig. 14*).

$$dw = 2\pi w a \sin\phi \sqrt{a^2\cos^2\phi + b^2\sin^2\phi} \times d\phi$$
$$= 2\pi a^2 w \sin\phi \sqrt{1 - \frac{a^2 - b^2}{a^2}\sin^2\phi} \times d\phi$$

Setting $k^2 = \frac{a^2 - b^2}{a^2}$ gives $dw = 2\pi a^2 w \sin\phi \sqrt{1 - k^2\sin^2\phi} \times d\phi$

Total weight, W, of area of revolution between Points 0 and 1 equals

$$W = \int dw = 2\pi a^2 w \int_{\phi=0}^{\phi} \sin\phi \sqrt{1 - k^2\sin^2\phi} \times d\phi^*.$$

$$W = 2\pi a^2 w \times$$
$$\left[-\frac{1}{2}\cos\phi\sqrt{1 - k^2\sin^2\phi} - \frac{1-k^2}{2k}\log\left(k\cos\phi + \sqrt{1-k^2\sin^2\phi}\right)\right]_{\phi=0}^{\phi}$$

Set $\cos\phi = \frac{y}{b} = g$ and $\sin^2\phi = 1 - \cos^2\phi = 1 - g^2$

$$W = 2\pi a^2 w\left[\frac{1}{2} + \frac{1 - k^2}{2k}\log\,(1 + k) - \frac{g}{2}\sqrt{1 - k^2(1-g^2)} - \frac{1 - k^2}{2k}\log\left(gk + \sqrt{1 - k^2\,(1 - g^2)}\right)\right]$$

Denote the quantity within the square bracket by C, then

$$W = 2\pi a^2 w C \qquad (17)$$

Values of C are presented in the table in terms of $\frac{y}{b}$ and $\frac{b}{a}$.

Derivations for meridional thrust, T, and hoop force, H, follow the same general procedure as illustrated for other domes.

Set W equal to the sum of vertical components of T, $W = 2\pi x T \sin\theta$

$x = a\sin\phi = a\sqrt{1 - g^2}$ (*see above*). For angle θ, see Fig. 15.

$$\tan\theta = \frac{dy}{dx} = -\frac{b\sin\phi\, d\phi}{a\cos\phi\, d\phi} = -\frac{b^2}{a^2} \times \frac{x}{y},$$

$$1 + \tan^2\theta = \frac{(a^2y)^2 + (b^2x)^2}{(a^2y)^2}$$

$$\sin\theta = \sqrt{\frac{\tan^2\theta}{1 + \tan^2\theta}} = \sqrt{\frac{b^4x^2}{(a^2y)^2 + (b^2x)^2}}$$
$$= \frac{b^2x}{\sqrt{a^4y^2 + b^4x^2}} = \frac{b\sqrt{1 - g^2}}{a\sqrt{1 - k^2(1 - g^2)}}$$

*For integration, see item 324 in *A Short Table of Integrals* by B. O. Pierce, (Third Edition) Ginn and Company, New York.

FIG. 15

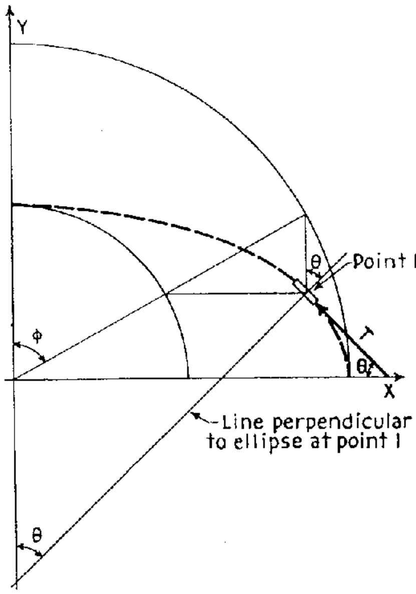
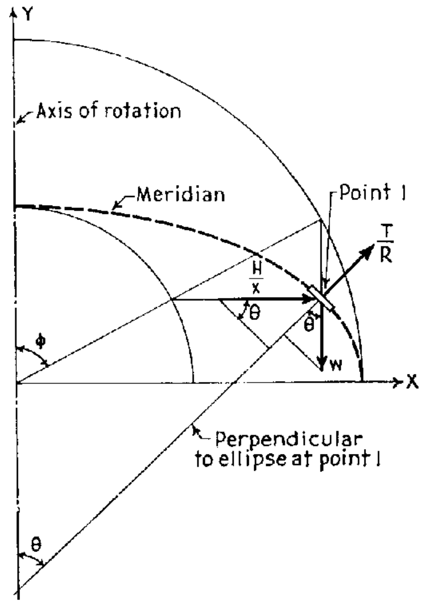


FIG. 16



$$T = \frac{W}{2\pi x \sin\theta} = W\frac{\sqrt{1-k^2(1-g^2)}}{2\pi b\,(1-g^2)} = a^2 wC\frac{\sqrt{1-k^2(1-g^2)}}{b(1-g^2)}$$

Set $Q = \sqrt{1-k^2(1-g^2)}$*. Values of Q are presented in the table.

$$T = \frac{wa^2}{b}\times\frac{CQ}{1-g^2} \tag{18}$$

Element of elliptical dome at Point 1 is subject to three forces, *see Fig. 16*:

(1) meridional thrust, T, tangential to meridian.
(2) hoop force, H, tangential to circle of latitude.
(3) load on element, w.

R is the radius of curvature and x is distance to axis of rotation.

Following the procedure used for spherical domes, we have

$$\frac{T}{R} + \frac{H}{x}\sin\theta - w\cos\theta = 0$$

$$\text{or } H = -\frac{Tx}{R\sin\theta} + \frac{wx\cos\theta}{\sin\theta} = -\frac{Wx}{2\pi x R\sin^2\theta} + wx\frac{a^2 y}{b^2 x}$$

$$= -\frac{W}{2\pi R\sin^2\theta} + \frac{wa^2 y^{**}}{b^2}$$

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}};\quad \frac{dy}{dx} = -\frac{b^2 x}{a^2 y};$$

$$\frac{d^2y}{dx^2} = -\frac{b^2}{a^2}\,\frac{y - x\frac{dy}{dx}}{y^2} = -\frac{b^2}{a^2}\,\frac{y + x\frac{b^2x}{a^2y}}{y^2} = -\frac{b^4}{a^2y^3}$$

$$R = \frac{\left(1+\frac{b^4x^2}{a^4y^2}\right)^{\frac{3}{2}}}{-\frac{b^4}{a^2y^3}} = \frac{-(a^4y^2+b^4x^2)^{\frac{3}{2}}\dagger}{b^4a^4}$$

$$R\sin^2\theta = \frac{(a^4y^2+b^4x^2)^{\frac{3}{2}}}{b^4a^4}\;\frac{b^4x^2}{a^4y^2+b^4x^2} = \frac{x^2}{a^4}\sqrt{a^4y^2+b^4x^2}$$

$$= b\left(\frac{x}{a}\right)^2\sqrt{\left(\frac{y}{b}\right)^2+\left(\frac{b}{a}\right)^2\left(\frac{x}{a}\right)^2} = b\,(1-g^2)\sqrt{1-k^2(1-g^2)}$$

$$H = -\frac{2\pi a^2 wC}{2\pi R\sin^2\theta} + \frac{wa^2}{b}\times\frac{y}{b}$$

$$= \frac{wa^2}{b}\left(g - \frac{C}{(1-g^2)\sqrt{1-k^2(1-g^2)}}\right);$$

set $Q = \sqrt{1-k^2(1-g^2)}$

$$\text{Therefore:}\quad H = \frac{wa^2}{b}\left(g - \frac{C}{(1-g^2)\,Q}\right) \tag{19}$$

Values of C and Q may be taken from table.

$$\text{At axis of rotation, } T = H = \frac{1}{2}wR = \frac{wa^2}{2b} \tag{20}$$

If w is considered the unit load per sq.ft. of horizontal projection instead of surface area, the following formulas apply.

$$\text{Total load: } W = w\pi x^2 = \pi a^2 w(1-g^2) \tag{21}$$

Meridional thrust:

$$T = \frac{W}{2\pi x\sin\theta} = \frac{wx}{2\sin\theta} = \frac{wa^2}{2b}\sqrt{1-k^2(1-g^2)}$$

$$= \frac{wa^2}{2b}Q \tag{22}$$

Hoop force:

$$H = -\frac{Tx}{R\sin\theta} + \frac{wx\cos^2\theta}{\sin\theta} = -\frac{wx^2}{2R\sin^2\theta} + \frac{wx\,(1-\sin^2\theta)}{\sin\theta}$$

$$= -\frac{wa^2(1-g^2)}{2b(1-g^2)\sqrt{1-k^2(1-g^2)}}$$

$$+\,wx\,\frac{a^2\,[1-k^2(1-g^2)] - b^2(1-g^2)}{a^2\,[1-k^2(1-g^2)]}\;\frac{a\sqrt{1-k^2(1-g^2)}}{b\sqrt{1-g^2}}$$

$$= -\frac{wa^2}{2b}\,\frac{1}{\sqrt{1-k^2(1-g^2)}} + \frac{wa^2}{2b}\,\frac{2g^2}{\sqrt{1-k^2(1-g^2)}}$$

$$= \frac{wa^2}{2b}\;\frac{2g^2-1}{Q} \tag{23}$$

*The quantity under the radical sign may also be written as:

$$g^2 + \frac{b^2}{a^2}(1-g^2)$$

**$\frac{\cos\theta}{\sin\theta}$ is positive and numerically equal to $\frac{1}{\tan\theta} = \frac{a^2y}{b^2x}$.

†R is actually a negative quantity, but we are here concerned with the numerical value only.

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