Design of Flexural Members subjected to Axial Force

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In this article the attention is drawn towards the design of flexural members subjected to axial force. It is shown that while using the charts of SP-16 the design not only becomes uneconomical but also unconservative.

There are cases when a flexural member is subjected to axial force; compression or tension. The members having such combined stresses are building frames (when subjected to lateral loads), arches, wall of a box section subjected to lateral forces, etc.

These flexural members should be designed for the axial force & moment as per basic equations derived here. If we use the charts of SP-16 for the columns subjected to uniaxial moment; we have to provide equal reinforcement on the both faces, because the charts have been prepared so & are primarily for compression members; obviously which is not economical.

\[
\begin{align*}
&f_c = 0.446 f_{ck} [2(\varepsilon_c/0.002) - (\varepsilon_c/0.002)^2] \quad \text{when } \varepsilon_c \leq 0.002 \\
&f_c = 0.446 f_{ck} \quad \text{when } 0.002 \leq \varepsilon_c \leq 0.0035
\end{align*}
\]

Area of the stress block = \(3x u f_c/7 + 8x u f_c/22\)

\(0.80952383x u f_c\)

where \(f_c\) can be equal to or less than \(0.446 f_{ck}\) depending upon the strain in the concrete.

Suppose the member is subjected to a moment \(M\) & a compressive force \(F\)

As total internal forces in the section are equal and opposite to the applied force

\(0.80952383 b x u f_c - A_{st} \sigma_{st} + A_{sc} \sigma_{sc} = F\)

As the depth of neutral axis can be represented as a fraction of effective depth \((=nd)\) this equation becomes

\(0.80952383 bnd f_c - A_{st} \sigma_{st} + A_{sc} \sigma_{sc} = F\)

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Taking the moment of the forces about c.g. of the tensile steel; we get

\[ 0.80952383 bnd f_c (d-0.416nd) + A_{sc} \sigma_{sc}(d-d') = M + F_g \]  

(2)

where \( g \) the distance between the center of gravity of the section & that of the tensile steel and \( d' \) is effective cover to the compression steel

From equation (2) we get stress in concrete as

\[ f_c = \frac{[M + F_g - A_{sc} \sigma_{sc}(d-d')]}{0.80952383 bnd f_c (d-0.416nd)} \]  

(3)

From equation (1) we get stress in steel as

\[ \sigma_{st} = \frac{[0.80952383 bnd f_c + A_{sc} \sigma_{sc} - F]}{A_{st}} \]  

(4)

We have \( n, A_{sc}, \) \& \( A_{st} \) as variables for a given section and forces. Obviously the design requires trial & error.

I made a program to solve these equations in Excel, using solver facility of the Excel. The program assumes the values of \( n, A_{sc}, \) \& \( A_{st} \) and first calculates the stress in concrete using eqn. (3); using this value of stress in concrete it calculates the stress in steel using eqn. (4).

The strain in steel is calculated as below

\[ \varepsilon_{st} = 0.002 + \frac{\sigma_{st}}{E_{st}} \]  

\[ E_{st} = 2 \times 10^5 \text{ Mpa} \]

For an assumed value of \( n \) & \( \varepsilon_{st} \) as above we calculate the strain in concrete as

\[ \varepsilon_{c} = \frac{\varepsilon_{st} n}{(1-n)} \]

We now calculate the stress in concrete corresponding to this value of strain using eqn. (a) or (b); depending upon the value of strain. If this value of stress in concrete comes out to be the same as one calculated by eqn. (3); the assumed value of variables is the correct.

This procedure also can be used for a section subjected to axial tension in addition to moment by putting negative value of \( F \).

**Example:** Suppose a section of 300mmX450mm (effective depth) is subjected to an ultimate moment of 150 Knm and ultimate axial compression 200 Kn. D (overall depth)= 500mm We will design it as per exact solution explained above and by the charts of SP-16.

Assume grade of concrete M20, grade of steel \( f_y \) 415

Solving this by above procedure we get

\[ n = 0.4791, \ p_c =0.1149\% \ (A_{sc} =155\text{mm}^2), \ p_t =0.6594\% \ (A_{st} =891\text{mm}^2) \]

\[ \text{total steel} = 1046 \text{ mm}^2 \]

Using SP-16

\[ Mu/f_{ck}bD^2 = 150 \times 10^6/(20*300*500^2) = 0.100 \]

\[ Pu/f_{ck}bD = 200 \times 10^3/(20*300*500) = 0.0667 \]

\[ d' = 50 \text{mm} \]

\[ p/f_{ck} = 0.05 \text{ hence } p = 1.0 \text{ %} \]
Total Area = 1500 mm$^2$, hence on tension side = 750 mm$^2$ < 891 mm$^2$

Hence we see that while we provide more total area by using SP-16 charts & in turn being unconservative on tension side; as the area provided on tension side is less than the actually required.

As per SP-16; for the points below the line $f_{st} = f_{yd}$ the outermost tension reinforcement undergoes inelastic deformation i.e. the stress in the reinforcement is more than the design yield strength ($0.87f_y$).

Closing Remarks

It is concluded that the design of flexural members with axial force should be carried out by basic principles not by any charts; in which equal reinforcement is provided on both faces; which is not economical for flexural members & sometimes unconservative too.