Effective Stiffness for Modeling Reinforced Concrete Structures

A Literature Review

By John-Michael Wong, Ph.D., S.E., Angie Sommer, S.E., Katy Briggs, S.E., and Cenk Ergin, P.E.

Seismology Concrete Subcommittee of the Structural Engineers Association of Northern California (SEAONC)

Seismic building design has typically been based on results from conventional linear analysis techniques. This type of analysis is a challenge for the design of reinforced concrete because the material is composite and displays nonlinear behavior that is dictated by the complex interaction between its components – the reinforcing steel and the concrete matrix. Simplifying the behavior of reinforced concrete components, so they can be modeled using a linear-elastic analysis approach, is vital to our ability to effectively design reinforced concrete structures.

Modeling of concrete structural elements using linear analysis to extract a reasonable structural response typically involves modifying the stiffness of concrete structural elements. However, this method presents its challenges, including the following:

- Effective stiffness is a function of the applied loading and detailing of the component. Reinforced concrete components behave differently under different loading conditions (e.g. tension, compression, flexure), as well as different rates of loading (impact, short term, long term).
- Applying stiffness modifiers can be an iterative process since the assumed stiffness of reinforced concrete elements in a structural analysis model influences the dynamic characteristics of the structure, which, in turn, changes the results of the analysis and the effective stiffness.
- Schedule demands pressure engineers to simplify the design process further, leading to only one stiffness modifier per element type applied to many analytical elements. This may be significantly inaccurate for a number of reasons, including:
    - o Analysis models can be very sensitive to the stiffness of a single element, (e.g. backstay effects due to at-grade concrete diaphragms or stiff podium structures in a tall building).
    - o Certain types of elements may have varying stiffnesses due to loading and location. For example, a multi-story column in a tall building will have a higher stiffness at the base compared to the roof.
    - o The design may warrant the consideration of multiple ground motion return periods, such as a service-level earthquake and a Maximum Considered Event (MCE) earthquake, each with a unique set of stiffness properties.

This article aids the structural engineer by providing a summary of the range of stiffness modifiers recommended by domestic and international publications for a variety of building components. A literature review of codes, standards, and research articles is provided, along with a brief summary of the key assumptions made in each document. Effective stiffness parameters for flexural and shear stiffness are summarized in the *Table* for easy comparison.

Domestic Codes

A summary of a variety of documents, which were published domestically and are typically used by structural engineers in the United States, is included below. Note that the recommendations provided in each document correlate to specific return periods or hazard events, or specific levels of applied loading. Some recommendations are independent of loading.

ACI 318, Building Code Requirements for Structural Concrete

ACI 318-11 is referenced by the 2012 *International Building Code* (IBC). Sections 8.8.1 through 8.8.3 provide guidelines for effective stiffness values to be used to determine deflections under lateral loading. In general, 50% of the stiffness based on gross section properties can be utilized for any element, or stiffness can be calculated in accordance with Section 10.10.4.1. ACI 318-14 contains similar recommendations for stiffness modifiers reformatted in Section 6.6.3.

Section 10.10.4, Elastic Second Order Analysis, provides both a table of effective stiffness values independent of load level and equations to derive stiffness based on loading and member properties. *Commentary* Section R10.10.4.1 explains that these recommendations are based on a series of frame tests and analyses, and include an allowance for the variability of computed deflections (MacGregor and Hage, 1977).

ASCE/SEI 41-13, Seismic Evaluation and Retrofit of Existing Buildings

Table 10-5 of ASCE 41-13 provides effective stiffness values to be used with linear procedures. Section 10.3.1.2.1 states that these may be used instead of computing the secant value to the yield point of the component, which is independent of the force level applied to the component.

ASCE 41 differentiates between columns with an axial load greater or less than $0.1*Ag*f'_c$ and refers to Elwood and Eberhard (2009) for further guidance regarding calculation of the effective stiffness of reinforced concrete columns.

Future editions of ASCE 41 will use ACI 369 as the source document for concrete buildings. The next revision, ACI 369-17, is anticipated to be published with ASCE 41-17 and will include improved stiffness provisions based on current research.

Table of stiffness assumptions for modeling concrete structures.

	Elements	Property Modifier for Modeling Elements												
		ACI 318-11 10.10.4.1 ACI 318-14 6.6.3.1.1	ASCE 41-13 Table 10-5	PEER TBI Guidelines Service Level	LATBSDC MCE-Level Non Linear Models (2014)	LATBSDC Servicability & Wind (2014)	FEMA 356 Table 6-5	NZS 3101: Part 2:2006 Ultimate Limit State (fy=300Mpa)	NZS 3101: Part 2:2006 Servicability Limit State (μ=3) (Note 3)	CSA A23.3-14	EuroCode	TS 500-2000	Paulay & Priestley (1992)	Priestly, Calvi & Kowalsky (2007)
Beams	Conventional Beams (L/H > 4)	0.35Ig	0.30Ig	0.50Ig	0.35Ig	0.70Ig	0.50Ig	0.40Ig (rectangular) 0.35Ig (T and L beams)	0.70Ig (rectangular) 0.60Ig (T and L beams)	0.35Ig	0.50Ig	0.40Ig	0.40Ig	0.17Ig-0.44Ig
	Prestressed Beams (L/H > 4)	n/a	1.00Ig	1.00Ig	n/a	n/a	1.00Ig	n/a	n/a				n/a	n/a
	Coupling Beams (L/H ≤ 4)		n/a	n/a	0.20Ig	0.30Ig	n/a	0.60Ig (diagonally reinforced)	0.75Ig				(9)	n/a
Columns	Columns - Pu ≥ 0.5Agf'c	0.70Ig	0.70Ig	0.50Ig	0.70Ig	0.90Ig	0.70Ig	0.80Ig	1.00Ig	0.70Ig	0.50Ig	0.80Ig (Note 6)	0.80Ig	0.12Ig-0.86Ig
	Columns - Pu ≤ 0.3Agf'c						0.50Ig	0.55Ig	0.80Ig				0.60Ig	
	Columns - Pu ≤ 0.1Agf'c		0.30Ig		n/a	n/a		0.40Ig	0.70Ig			0.40Ig	(9)	
	Columns - tension			n/a				n/a	n/a			n/a		
Walls (4)	Walls - uncracked	0.70Ig	n/a	0.75Ig	n/a	n/a	0.80Ig	n/a	n/a	0.7Ig	0.50Ig	n/a	(9)	n/a
	Walls - cracked	0.35Ig	0.50Ig		1.00Ec (1)	0.75Ig	0.50Ig	0.32Ig-0.48Ig	0.50Ig-0.70Ig	0.35Ig	0.50Ig	0.40Ig - 0.80Ig (Note 6)		0.20Ig-0.30Ig
	Walls - shear	n/a	0.40EcAw (10	n/a	0.50Ag	1.00Ag	n/a	n/a	n/a	n/a	n/a	n/a	(9)	n/a
Slabs	Conventional flat plates and flat slabs	0.25Ig	See 10.4.4.2	0.50Ig	0.25Ig	0.50Ig	n/a	n/a	n/a	0.25Ig	0.50Ig	n/a	(9)	n/a
	Post tensioned flat plates and flat slabs	n/a	See 10.4.4.2									n/a	n/a	
	In-plane Shear	n/a	n/a	n/a	0.25Ag	0.80Ag	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
Notes		(5)	(2)	(2)	(2)				(3)				(7)	

Notes

(1) Non-linear fiber elements automatically account for cracking of concrete because the concrete fibers have zero tension stiffness.

(2) Elastic modulus may be computed using expected material strengths.

(3) μ is ductility capacity.

(4) Wall stiffness is intended for in-plane wall behavior.

(5) ACI 318-11 Section 8.8 (ACI 318-14, Section 6.6) permits the assumption of 0.50Ig for all elements under factored lateral load analysis.

(6) TS 500-2000 specifies the use of 0.4Ig for Pu/Ac/f'c < 0.1 and the use of 0.8Ig for Pu/Ac/f'c > 0.4; interpolate for all values in between 0.1 and 0.4.

(7) T and L beams should use recommended values of 0.35 Ig. For columns, categories are P = 0.2 f'c Ag and P = -0.05 f'c Ag

(8) Shear stifness properties are unmodified unless specifically noted otherwise.

(9) Effective stiffness per equation. See reference for more information.

(10) Note that G = 0.4*I, so ASCE 41-13 is recommending that a modifier of 1.0 be used for the shear stiffness of concrete shear walls; that is, they recommmend no reduction in shear stiffness.

Definitions

Ig = Gross moment of inertia
L = Clear span of coupling beam
H = Height of coupling beam
Pu = Factored axial load
Ag = Ac = Gross (uncracked) area
f'c = Compressive strength of concrete
Ec = Modulus of elasticity of concrete
fy = Yield stress of reinforcing steel
MPa = Megapascals
Aw = Horizontal area

PEER Tall Buildings Initiative

Guidelines for Performance-Based Seismic Design of Tall Buildings, also referred to as the Tall Buildings Initiative (TBI), is a consensus document that presents a recommended alternative to the prescriptive procedures for the seismic design of buildings taller than 160 feet. Whereas prescriptive requirements suggest a dual system, the alternative procedures in TBI allow for the use of shear-wall-only structures.

While much of the PEER TBI document focuses on nonlinear analysis for larger earthquakes, the provisions of this document also give a set of recommendations for effective component stiffness values to use in a linear-elastic model subjected to a service-level earthquake (minimum return period of 43 years or 50% probability of exceedance in 30 years). The provisions of this document are meant to apply only to relatively slender structures with long fundamental vibration periods, and with significant mass participation and lateral response in higher modes of vibration.

Los Angeles Tall Buildings Structural Design Council (LATBSDC) Manual

Section 2.5 requires structural models to incorporate realistic estimates of stiffness

and strength considering the anticipated level of excitation and damage. In lieu of a detailed analysis, the effective reinforced concrete stiffness properties given in Table 3 of that document may be used. This table provides separate values for MCE-level seismic event nonlinear models as opposed to serviceability seismic events and wind loads. A serviceability seismic event is defined to have 50% probability of exceedance in 30 years; the MCE-level event is equivalent to the MCE_R of ASCE 7-10, which has a 2% probability of exceedance in 50 years. *Commentary* Section C.3.2.4 also states that stiffness properties may be derived from test data or from Moehle et al. (2008).

International Codes and Other References

A summary of a variety of documents published outside of the United States, is included below. Note that the recommendations provided in each document correlate to specific return periods or hazard events, or specific levels of applied loading, and some recommendations are independent of loading.

New Zealand Standard

NZS 3101: Part 2 (2006 Edition) states that effective stiffness in concrete members is influenced by the amount and distribution of reinforcement, the extent of cracking, tensile strength of the concrete, and initial conditions in the member before structural actions are applied.

To simplify the complex analysis that would be required to address these factors, the standard lists recommended effective stiffnesses for different members, similar to U.S. codes. However, the level of loading used in NZS 3101 differs from U.S. codes. The ultimate limit state earthquake for a typical structure (importance level 2) is based on a 10% probability of exceedance in 50 years for a structure with a 50-year design life. The ultimate limit state earthquake for a structure with an importance level of 4 is based on a 2% probability of exceedance in 50 years. The serviceability limit state earthquake for all structures is based on an annual probability of exceedance equal to one in 25 for a structure with a 50-year design life.

Canadian Standards Association Design of Concrete Structures

CSA A23.4-14 provides recommended stiffness modification factors in Section 10.14.1.2. These factors are provided to determine the first-order lateral story deflections based on an elastic analysis. The Canadian Standards are based on an earthquake with a 2% probability of exceedance in 50 years.

European Codes

According to Eurocode 8 (EN1998-3), the elastic stiffness of the bilinear force-deformation relation in reinforced concrete elements should correspond to that of cracked sections and the initiation of yielding of the reinforcement. Unless a more accurate analysis of the cracked elements is performed, this standard recommends that the elastic flexural and shear stiffness properties of concrete elements are taken as 50% of the corresponding stiffness of the uncracked element.

Part 3 of Eurocode 8 provides an equation based on moment-to-shear ratio and yield rotation, which can be used for determination of a more accurate effective stiffness. Both ultimate level and serviceability level loads are addressed in Eurocode 8 for linear and nonlinear analysis.

Turkish Standard

Turkish TS 500-2000 refers to the *Turkish Earthquake Code* (2007), which states that uncracked properties shall be used for components when performing certain types of analyses. However, stiffness modifiers for cracked section properties may be utilized for beams framing into walls in their own plane and for coupling beams of coupled structural walls when performing these types of analyses. Cracked section properties must be used for the analysis of existing structures. Cracked section properties may also be used when performing advanced analyses.

Paulay and Priestley (1992), Seismic Design of Reinforced Concrete and Masonry Buildings

Paulay and Priestley provide recommendations for stiffness modifiers for cracked concrete frame members and shear walls. In their discussion of stiffness modifiers for frame members, they emphasize the inherent approximation in the use of stiffness modifiers.

Recommendations for frame stiffness are provided in Table 4.1 (Pauley and Priestley). The authors note that the column stiffness should be a function of the axial load, with the permanent gravity load taken as 1.1 times the dead load plus the axial load resulting from seismic overturning effects. For the analysis of concrete wall structures, the authors recommend the use of component-specific equations to determine their effective stiffness.

Priestley, Calvi, and Kowalsky (2007), Displacement-Based Seismic Design

Priestley, Calvi, and Kowalsky conclude that the stiffness of a member is related to its strength, and that yield curvature is independent of strength. Because of the strength-stiffness relationship, they recommend that engineers performing force-based analyses should always treat the assignment of stiffness modifiers as an iterative process.

This reference provides ranges of stiffness modifiers based on different member strengths for various reinforced concrete elements, all of which correspond to displacement-based seismic design. However, the authors assume that these recommendations can be used for force-based seismic design as long as an iterative process is used.

Conclusion

As shown in the *Table* (*page 19*) and discussed above, different standards and codes provide varying guidelines for modifying the stiffness of reinforced concrete elements. When performing a structural analysis, it is useful to review multiple codes and standards to determine the effective stiffnesses of elements. The information derived from multiple sources may reveal a more accurate method of analysis for the particular structure the designer is currently assessing. Because the effective stiffnesses of reinforced concrete elements can have significant effects on the results of structural analysis, it is prudent for the designer to understand the appropriate modification factors and, in some cases, run multiple analyses using upper- and lower-bound stiffness modification factors.▪

John-Michael Wong, Ph.D., S.E., is an Associate at KPFF in San Francisco, California. He has served on the SEAONC Concrete Subcommittee since 2014 and can be reached at ***john-michael.wong@kpff.com****.*

Angie Sommer, S.E., is an Associate at ZFA Structural Engineers in San Francisco, California. She has served on the SEAONC Concrete Subcommittee since 2014 and can be contacted at ***angiesommer@gmail.com****.*

Katy Briggs, S.E., is a Principal at BASE Design in San Francisco, California. She is the current chair of the SEAONC Concrete Subcommittee and the vice-chair of the SEAONC Seismology Committee. She can be reached at ***katy@basedesigninc.com****.*

Cenk Ergin, P.E., is a Senior Design Engineer at Gilbane in Concord, California. He is past chair of the SEAONC Concrete Subcommittee. He can be contacted at ***cergin@gilbaneco.com****.*

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