Example hand calculations for local and distortional buckling stress of a simple lipped channel column.

Calculations for:
1. Flange Local Buckling (k=4 solution)
2. Web Local Buckling (k=4 solution)
3. Lip Local Buckling (k=0.43 solution)
4. Flange/Lip Local Buckling (Schafer 1997)
5. Flange/Web Local Buckling (Schafer -unpublished)
6. Distortional buckling (Schafer 1997*)
7. Distortional buckling (Lau and Hancock 1987**)
8. AISI edge stiffened element via AISI (1996)

*with corrections, given below, July 1998.
** with corrections given below, January 1999

Specimen Dimensions and Properties:

$h := 2.5$

$b := 1.328$

$d := 0.328$

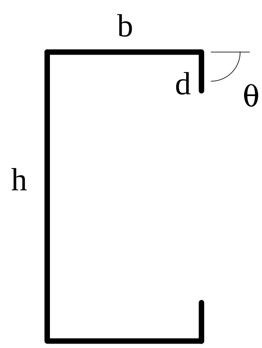
$\theta := 90 \cdot \frac{\pi}{180}$

$t := 0.0284$

$E := 29500$

$\nu := 0.3$

$f := 50$



Glossary of Variables:
h = web height
b = flange width
d = lip length
θ = lip angle (radians)
t = thickness
E = Young's modulus
ν = Poisson's ratio
f = compressive stress (necessary for AISI only)

Local Buckling Element Models: Each element is treated separately

1. Flange Local Buckling:

Classical solution for a simply supported plate in pure compression is employed.

$k_{flange} := 4$

$$f_{cr\_flange} := k_{flange} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_flange} = 48.775$$

2. Web Local Buckling:

Classical solution for a simply supported plate in pure compression is employed.

$k_{web} := 4$

$$f_{cr\_web} := k_{web} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{h}\right)^2 \qquad f_{cr\_web} = 13.763$$

3. Lip Local Buckling:

Classical solution for a plate simply supported on three sides and free along one edge is employed.

$k_{lip} := 0.43$

$$f_{cr\_lip} := k_{lip} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{d}\right)^2 \qquad f_{cr\_lip} = 85.952$$

Note, the local buckling stress of the member can conservatively be predicted by taking the minimum of 1, 2 and 3. In some cases, this calculation will be very conservative since it completely ignores any interaction of the elements.

Local Buckling Interaction Models

4. Flange / Lip Local Buckling

This expression for k, was derived in Schafer (1997). The expression is based on an empirical curve fit to finite strip analysis of an isolated flange and lip. The expression accounts for the beneficial affect of the lip on the flange at intermediate lip lengths and also accounts for the detrimental affect of the lip on the flange at long lip lengths.

$$k_{flange\_lip} := -11.07 \cdot \left(\frac{d}{b}\right)^2 + 3.95 \cdot \left(\frac{d}{b}\right) + 4 \qquad k_{flange\_lip} = 4.3$$

Note, d/b should be less than 0.6 for this empirical expression. A more general expression for cases when the unstiffened element is under a stress gradient and the edge stiffened element is in pur compression (i.e. the flange of a flexural member) can be found in Schafer (1997).

$$f_{cr\_flange\_lip} := k_{flange\_lip} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_flange\_lip} = 52.437$$

5. Flange / Web Local Buckling

This expression is newly derived for this work. The expression is based on an empirical curve fit to finite strip analysis of an isolated flange and web. If h/b = 1 The k value is 4. If h/b > 1 the k value is reduced from 4 due to the buckling of the web. If h/b < 1 the k value is increased from 4 due to the restraint provided by the web to the flange.

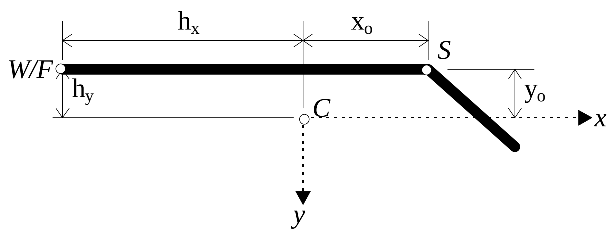
$$k_{flange\_web} := \begin{cases} \left[\left[2 - \left(\frac{b}{h}\right)^{0.4}\right] \cdot 4 \cdot \left(\frac{b}{h}\right)^2\right] & \text{if } \frac{h}{b} \geq 1 \\ \left[\left[2 - \left(\frac{h}{b}\right)^{0.2}\right] \cdot 4\right] & \text{if } \frac{h}{b} < 1 \end{cases} \qquad k_{flange\_web} = 1.381$$

$$f_{cr\_flange\_web} := k_{flange\_web} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_flange\_web} = 16.84$$

Note, the local buckling stress of the member can be taken as the minimum of 4 and 5. This provides a good estimate of the actual member local buckling stress.

6 and 7: Flange Properties for use in the Distortional Buckling Calculation

The hand methods for distortional buckling prediction require that section properties of the isolated flange be calculated. The expressions here are only applicable for simple lips. More complicated flanges would follow the same procedures, but new expressions would be required



Material Properties:

$$G := \frac{E}{2\cdot(1+\nu)}$$

Properties of the Flange Only:

$$A := (b+d)\cdot t \qquad A = 0.047$$

$$J := \frac{1}{3}\cdot b\cdot t^3 + \frac{1}{3}\cdot d\cdot t^3 \qquad J = 1.264\cdot 10^{-5}$$

$$I_x := \frac{t\cdot\left(t^2\cdot b^2 + 4\cdot b\cdot d^3 - 4\cdot b\cdot d^3\cdot\cos(\theta)^2 + t^2\cdot b\cdot d + d^4 - d^4\cdot\cos(\theta)^2\right)}{12\cdot(b+d)} \qquad I_x = 2.87\cdot 10^{-4}$$

$$I_y := \frac{t\cdot\left(b^4 + 4\cdot d\cdot b^3 + 6\cdot d^2\cdot b^2\cdot\cos(\theta) + 4\cdot d^3\cdot b\cdot\cos(\theta)^2 + d^4\cdot\cos(\theta)^2\right)}{12\cdot(b+d)} \qquad I_y = 8.836\cdot 10^{-3}$$

$$I_{xy} := \frac{t\cdot b\cdot d^2\cdot\sin(\theta)\cdot(b + d\cdot\cos(\theta))}{4\cdot(b+d)} \qquad I_{xy} = 8.135\cdot 10^{-4}$$

$$I_o := \frac{t\cdot b^3}{3} + \frac{b\cdot t^3}{12} + \frac{t\cdot d^3}{3} \qquad I_o = 0.023$$

$$x_o := \frac{b^2 - d^2\cdot\cos(\theta)}{2\cdot(b+d)}$$ x distance from the centroid to the shear center. $x_o = 0.532$

$$y_o := \frac{-d^2\cdot\sin(\theta)}{2\cdot(b+d)}$$ y distance from the centroid to the shear center. $y_o = -0.032$

$$h_x := \frac{-\left(b^2 + 2\cdot d\cdot b + d^2\cdot\cos(\theta)\right)}{2\,(b+d)}$$ x distance from the centroid to the web/flange juncture. $h_x = -0.796$

$$h_y := \frac{-d^2\cdot\sin(\theta)}{2\cdot(b+d)}$$ y distance from the centroid to the web/flange juncture. $h_y = -0.032$

$$C_w := 0 \qquad C_w = 0$$

6. Distortional Buckling (Schafer 1997)

Determine the critical half-wavelength at which distortional buckling occurs:

$$L_{cr} := \left[\frac{6\cdot\pi^4\cdot h\cdot\left(1-\nu^2\right)}{t^3}\cdot\left[I_x\cdot\left(x_o-h_x\right)^2+C_w-\frac{I_{xy}^{\;2}}{I_y}\cdot\left(x_o-h_x\right)^2\right]\right]^{\frac{1}{4}} \qquad L_{cr}=12.139$$

If bracing is provided that restricts the distortional mode at some length less than Lcr, then this length should be used in place of Lcr.

Determine the elastic and "geometric" rotational spring stiffness of the flange:

$$k_{\phi fe} := \left(\frac{\pi}{L_{cr}}\right)^4\cdot\left[E\cdot I_x\cdot\left(x_o-h_x\right)^2+E\cdot C_w-E\cdot\frac{I_{xy}^{\;2}}{I_y}\cdot\left(x_o-h_x\right)^2\right]+\left(\frac{\pi}{L_{cr}}\right)^2\cdot G\cdot J$$

$$k_{\phi fe}=0.059$$

$$k_{\phi fg} := \left(\frac{\pi}{L_{cr}}\right)^2\cdot\left[A\cdot\left[\left(x_o-h_x\right)^2\cdot\left(\frac{I_{xy}}{I_y}\right)^2-2\cdot y_o\cdot\left(x_o-h_x\right)\cdot\left(\frac{I_{xy}}{I_y}\right)+h_x^{\;2}+y_o^{\;2}\right]+I_x+I_y\right]$$

$$k_{\phi fg}=2.68\bullet 10^{-3}$$

Determine the elastic and "geometric" rotational spring stiffness from the web:

$$k_{\phi we} := \frac{E\cdot t^3}{6\cdot h\cdot\left(1-\nu^2\right)} \qquad k_{\phi we}=0.05$$

$$k_{\phi wg} := \left(\frac{\pi}{L_{cr}}\right)^2\cdot\frac{t\cdot h^3}{60} \qquad k_{\phi wg}=4.954\bullet 10^{-4}$$

$k\phi wg$ is modified due to an error in Schafer (1997) analysis.

Determine the distortional buckling stress:

$$f_{cr\_dist\_schafer} := \frac{k_{\phi fe}+k_{\phi we}}{k_{\phi fg}+k_{\phi wg}}$$

$$f_{cr\_dist\_schafer}=34.205$$

7. Lau and Hancock (1987) Formulation

The notation for Lau and Hancock (1987) is slightly different than in the previous approach. The original notation is employed to aid comparisions to Lau and Hancock (1987).

$x_{bar} := b - x_o$ $\quad x_{bar} = 0.796$ $\qquad y_{bar} := -y_o$ $\quad y_{bar} = 0.032$

The critical half-wavelength for distortional buckling λd is first estimated

$$\lambda_d := 4.80 \cdot \left(\frac{I_x \cdot b^2 \cdot h}{t^3}\right)^{\frac{1}{4}} \qquad \lambda_d = 13.086$$

The next step is to estimate the distortional buckling stress. This estimate is required, because the rotational stiffness is written as a function of the distortional buckling stress. This step requires formulation and solution of a quadratic equation.

Parameters required for the solution:

$$\eta := \left(\frac{\pi}{\lambda_d}\right)^2 \qquad \beta_1 := x_{bar}^{\,2} + \left(\frac{I_x + I_y}{A}\right) \qquad \beta_1 = 0.827$$

$$\alpha_1 := \frac{\eta}{\beta_1} \cdot \left(I_x \cdot b^2 + 0.039 \cdot J \cdot \lambda_d^{\,2}\right) \quad \alpha_2 := \eta \cdot \left(I_y + \frac{2}{\beta_1} \cdot y_{bar} \cdot b \cdot I_{xy}\right) \quad \alpha_3 := \eta \cdot \left(\alpha_1 \cdot I_y - \frac{\eta}{\beta_1} \cdot I_{xy}^{\,2} \cdot b^2\right)$$

$$\alpha_1 = 4.117 \cdot 10^{-5} \qquad \alpha_2 = 5.142 \cdot 10^{-4} \qquad \alpha_3 = 1.628 \cdot 10^{-8}$$

The solution to the quadratic has two roots, which are found as:

$$root_{pos}(E, A, \alpha_1, \alpha_2, \alpha_3) := \frac{E}{2 \cdot A} \cdot \left[(\alpha_1 + \alpha_2) + \left[(\alpha_1 + \alpha_2)^2 - 4 \cdot \alpha_3\right]^{\frac{1}{2}}\right]$$

$$root_{neg}(E, A, \alpha_1, \alpha_2, \alpha_3) := \frac{E}{2 \cdot A} \cdot \left[(\alpha_1 + \alpha_2) - \left[(\alpha_1 + \alpha_2)^2 - 4 \cdot \alpha_3\right]^{\frac{1}{2}}\right]$$

The smaller of the two roots is of interest. In this case $root_{neg}$ is used.
In cases where the root is negative the distortional buckling stress is set to zero.

note:

$$root_{pos}(E, A, \alpha_1, \alpha_2, \alpha_3) = 328.887$$

$$root_{neg}(E, A, \alpha_1, \alpha_2, \alpha_3) = 19.472$$

$$f'_{ed} := \max\left(\left[root_{neg}(E, A, \alpha_1, \alpha_2, \alpha_3) \quad 0\right]\right) \qquad f'_{ed} = 19.472$$

(estimated dist. stress, used to estimate $k\phi$)

Now that the distortional buckling stress has been estimated, the rotational stiffness may be determined:

$$k_{\phi} := \frac{E \cdot t^3}{5.46 \cdot \left(h + 0.06 \cdot \lambda_d\right)} \cdot \left[1 - \frac{1.11 \cdot f'_{ed}}{E \cdot t^2} \cdot \left(\frac{h^2 \cdot \lambda_d}{h^2 + \lambda_d^{\,2}}\right)^2\right]$$

$$k_{\phi} = 0.03$$

This rotational stiffness is roughly equivalent to the web elastic + web geometric stiffness mentioned in Schafer (1997). If the geometric stiffness of the web is greater than the elastic stiffness of the web, a negative $k\phi$ will result. This does not necessarily imply buckling ensues, because the elastic stiffness of the flange may be great enough to overcome the web contribution.

The original Lau and Hancock (1987) model for columns was updated in Hancock et al. (1996) for beams. The update treats the $k\phi<0$ and the $k\phi>0$ as two different cases. The older model for columns is employed here.

Calculation of the buckling stress:

$$f_{ed} := \left| \begin{array}{l} \alpha_1 \leftarrow \frac{\eta}{\beta_1} \cdot \left(I_x \cdot b^2 + 0.039 \cdot J \cdot \lambda_d^{\,2}\right) + \frac{k_{\phi}}{\beta_1 \cdot \eta \cdot E} \\ \alpha_3 \leftarrow \eta \cdot \left(\alpha_1 \cdot I_y - \frac{\eta}{\beta_1} \cdot I_{xy}^{\,2} \cdot b^2\right) \\ root1 \leftarrow root_{pos}\left(E, A, \alpha_1, \alpha_2, \alpha_3\right) \\ root2 \leftarrow root_{neg}\left(E, A, \alpha_1, \alpha_2, \alpha_3\right) \\ \max((root2 \quad 0)) \end{array} \right.$$

Note that in cases where the negative root is less than zero the distortional buckling stress is set to zero. This is consistent with the approach of Lau and hancock (1987) as employed in the joint Australian/New Zealand standard.

The final result is:

$$f_{cr\_dist\_hancock} := f_{ed}$$

$$f_{cr\_dist\_hancock} = 32.607$$

8. AISI (1996) Calculation for an edge stiffened element

Preliminaries: $S := 1.28 \cdot \sqrt{\frac{E}{f}}$ $I_s := \frac{t \cdot d^3 \cdot \sin(\theta)^2}{12}$

$$
k_{aisi} := \left|
\begin{array}{l}
4 \ \text{ if } \frac{b}{t} \le \frac{S}{3} \\
\text{if } \frac{S}{3} < \frac{b}{t} \le S \\
\quad \left|
\begin{array}{l}
k_u \leftarrow 0.43 \\
I_a \leftarrow t^4 \cdot 399 \cdot \left[\frac{\frac{b}{t}}{S} - \left(\frac{k_u}{4}\right)^{\frac{1}{2}} \right]^3 \\
n \leftarrow \frac{1}{2} \\
C2 \leftarrow \min\left(\begin{bmatrix} \frac{I_s}{I_a} & 1 \end{bmatrix} \right) \\
C1 \leftarrow 2 - C2 \\
k_a \leftarrow \min\left(\begin{bmatrix} 5.25 - 5 \cdot \frac{d}{b} & 4 \end{bmatrix} \right) \\
C2^n \cdot (k_a - k_u) + k_u
\end{array}
\right. \\
\text{if } \frac{b}{t} \ge S \\
\quad \left|
\begin{array}{l}
k_u \leftarrow 0.43 \\
I_a \leftarrow t^4 \cdot \left(115 \cdot \frac{\frac{b}{t}}{S} + 5 \right) \\
n \leftarrow \frac{1}{3} \\
C2 \leftarrow \min\left(\begin{bmatrix} \frac{I_s}{I_a} & 1 \end{bmatrix} \right) \\
C1 \leftarrow 2 - C2 \\
k_a \leftarrow \min\left(\begin{bmatrix} 5.25 - 5 \cdot \frac{d}{b} & 4 \end{bmatrix} \right) \\
C2^n \cdot (k_a - k_u) + k_u
\end{array}
\right.
\end{array}
\right.
$$

The AISI calculation for k is based on the slenderness of the flange. Different solutions for k are found depending on how slender the compression flange is. For instance, in case 1, k = 4 because the flange is stocky enough that all edge stiffeners are expected to be adequate.

This is the only k calculated for the flange and thus it accounts for both local and distortional buckling of the flange.

The final result is:

$$k_{aisi} = 3.632$$

$$f_{cr\_aisi} := k_{aisi} \cdot \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t}{b}\right)^2$$

$$f_{cr\_aisi} = 44.285$$

Summary Results finite strip values are for a 2.5x1.328x0.328x0.0284 lipped C

$f_{cr\_strip\_local} := 18.96$

$f_{cr\_strip\_distortional} := 32.64$

$f_{cr\_flange} = 48.775$

$f_{cr\_web} = 13.763$

$f_{cr\_lip} = 85.952$

$f_{cr\_flange\_lip} = 52.437$

$f_{cr\_flange\_web} = 16.84$

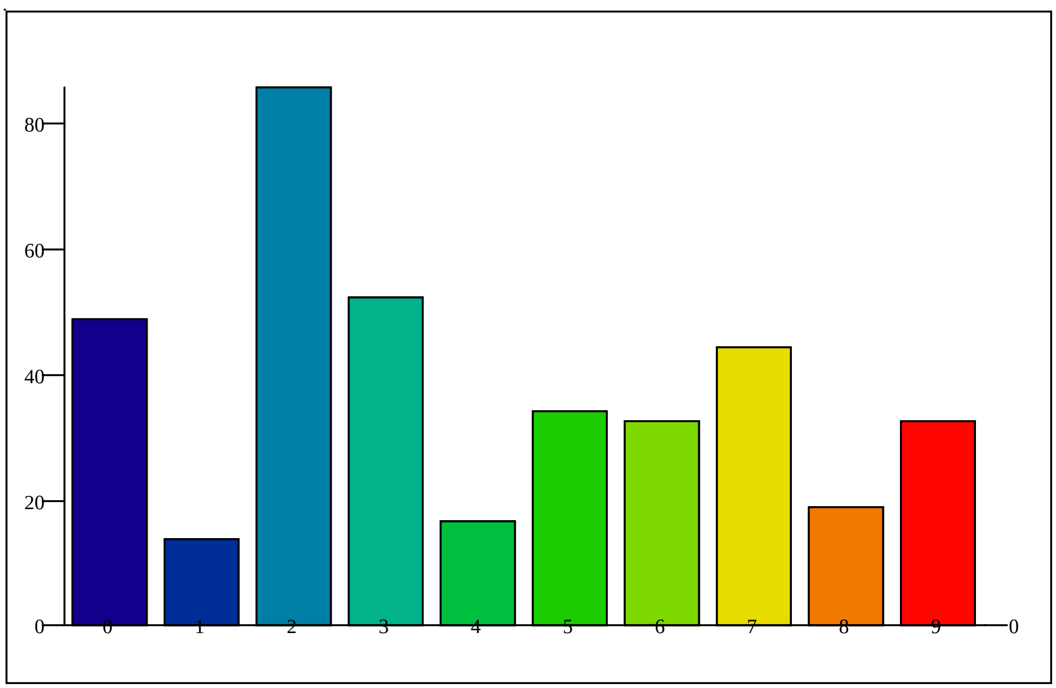
$f_{cr\_dist\_schafer} = 34.205$

$f_{cr\_dist\_hancock} = 32.607$

$f_{cr\_aisi} = 44.285$

This is the order of the plotted values:

$$f_{cr\_all} := \begin{bmatrix} f_{cr\_flange} \\ f_{cr\_web} \\ f_{cr\_lip} \\ f_{cr\_flange\_lip} \\ f_{cr\_flange\_web} \\ f_{cr\_dist\_schafer} \\ f_{cr\_dist\_hancock} \\ f_{cr\_aisi} \\ f_{cr\_strip\_local} \\ f_{cr\_strip\_distortional} \end{bmatrix}$$



$f_{cr\_all}^{T}$