11.1 INTRODUCTION

Corbel or bracket is a reinforced concrete member is a short-haunched cantilever used to support the reinforced concrete beam element. Corbel is structural element to support the pre-cast structural system such as pre-cast beam and pre-stressed beam. The corbel is cast monolithic with the column element or wall element.

This chapter is describes the design procedure of corbel or bracket structure. Since the load from pre-cast structural element is large then it is very important to make a good detailing in corbel.

11.2 BEHAVIOR OF CORBEL

The followings are the major items show the behavior of the reinforced concrete corbel, as follows:

- The **shear span/depth ratio is less than 1.0**, it makes the corbel behave in two-dimensional manner.
- **Shear deformation** is significant in the corbel.
- There is **large horizontal force** transmitted from the supported beam result from long-term shrinkage and creep deformation.
- **Bearing failure** due to large concentrated load.
- The cracks are usually **vertical or inclined pure shear cracks**.
- The mode of failure of corbel are: yielding of the tension tie, failure of the end anchorage of the tension tie, failure of concrete by compression or shearinga and bearing failure.

The followings figure shows the mode of failure of corbel.
11.3 SHEAR DESIGN OF CORBEL

11.3.1 GENERAL

Since the corbel is cast at different time with the column element then the cracks occurs in the interface of the corbel and the column. To avoid the cracks we must provide the shear friction reinforcement perpendicular with the cracks direction.

ACI code uses the shear friction theory to design the interface area.

11.3.2 SHEAR FRICTION THEORY

In shear friction theory we use coefficient of friction $\mu$ to transform the horizontal resisting force into vertical resisting force.

The basic design equation for shear reinforcement design is:

$$\phi V_n \geq V_u$$

[11.1]

where:

- $V_n$ = nominal shear strength of shear friction reinforcement
- $V_u$ = ultimate shear force
- $\phi$ = strength reduction factor ($\phi = 0.85$)
The nominal shear strength of shear friction reinforcement is:

$$V_n = A_{vf} f_y \mu$$

where:
- $V_n$ = nominal shear strength of shear friction reinforcement
- $A_{vf}$ = area of shear friction reinforcement
- $F_y$ = yield strength of shear friction reinforcement
- $\mu$ = coefficient of friction

### TABLE 11.1  SHEAR FRICTION REINFORCEMENT STRENGTH

<table>
<thead>
<tr>
<th></th>
<th>VERTICAL SHEAR FRICTION REINFORCEMENT</th>
<th>INCLINED SHEAR FRICTION REINFORCEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_n$</td>
<td>$A_{vf} = \frac{V_n}{F_y \mu}$</td>
<td>$A_{vf} = \frac{V_n}{f_y (\mu \sin \alpha_f + \cos \alpha_f)}$</td>
</tr>
<tr>
<td>$A_{vf}$</td>
<td>$\frac{V_n}{F_y \mu}$</td>
<td>$\frac{V_n}{f_y (\mu \sin \alpha_f + \cos \alpha_f)}$</td>
</tr>
</tbody>
</table>

The value of $\lambda$ is:
- $\lambda = 1.0$  normal weight concrete
- $\lambda = 0.85$ sand light weight concrete
- $\lambda = 0.75$ all light weight concrete
The ultimate shear force must follows the following conditions:

\[ V_u \leq \phi(0.2f'_c) b_w d \]
\[ V_u \leq \phi(5.5) b_w d \]

where:
- \( V_u \) = ultimate shear force (N)
- \( f'_c \) = concrete cylinder strength (MPa)
- \( b_w \) = width of corbel section (mm)
- \( d \) = effective depth of corbel (mm)

11.3.3 Step – By – Step Procedure

The followings are the step – by – step procedure used in the shear design for corbel (bracket), as follows:

- Calculate the ultimate shear force \( V_u \).
- Check the ultimate shear force for the following condition, if the following condition is not achieved then enlarge the section.

\[ V_u \leq \phi(0.2f'_c) b_w d \]
\[ V_u \leq \phi(5.5) b_w d \]

- Calculate the area of shear friction reinforcement \( A_{vf} \).

<table>
<thead>
<tr>
<th>VERTICAL SHEAR FRICTION REINFORCEMENT</th>
<th>INCLINED SHEAR FRICTION REINFORCEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_n )</td>
<td>( A_{vf} )</td>
</tr>
<tr>
<td>( V_n = A_{vf} f_y \mu )</td>
<td>( A_{vf} = \frac{V_n}{f_y \mu} )</td>
</tr>
</tbody>
</table>

- The design must be follows the basic design equation as follows:

\[ \phi V_n \geq V_u \]

11.4 Flexural Design of Corbel

11.4.1 General

The corbel is design due to ultimate flexure moment result from the supported beam reaction \( V_u \) and horizontal force from creep and shrinkage effect \( N_u \).
11.4.2 TENSION REINFORCEMENT

The ultimate horizontal force acts in the corbel $N_{uc}$ is result from the creep and shrinkage effect of the pre-cast or pre-stressed beam supported by the corbel. This ultimate horizontal force must be resisted by the tension reinforcement as follows:

$$A_n = \frac{N_{uc}}{4f_y}$$  \[11.2\]

where:

- $A_n$ = area of tension reinforcement
- $N_{uc}$ = ultimate horizontal force at corbel
- $f_y$ = yield strength of the tension reinforcement
- $\phi$ = strength reduction factor ($\phi = 0.85$)

Minimum value of $N_{uc}$ is $0.2V_{uc}$.

The strength reduction factor is taken 0.85 because the major action in corbel is dominated by shear.

11.4.3 FLEXURAL REINFORCEMENT
The ultimate flexure moment $M_u$ result from the support reactions is:

$$M_u = V_u(a) + N_{uc}(h - d) \quad [11.3]$$

where:
- $M_u$ = ultimate flexure moment
- $V_u$ = ultimate shear force
- $a$ = distance of $V_u$ from face of column
- $N_{uc}$ = ultimate horizontal force at corbel
- $h$ = height of corbel
- $d$ = effective depth of corbel

The resultant of tensile force of tension reinforcement is:

$$T_f = A_f f_y \quad [11.4]$$

where:
- $T_f$ = tensile force resultant of flexure reinforcement
- $A_f$ = area of flexure reinforcement
- $f_y$ = yield strength of the flexure reinforcement

The resultant of compressive force of the concrete is:

$$C_c = 0.85f'_c ba(\cos \beta) \quad [11.5]$$

where:
- $C_c$ = compressive force resultant of concrete
- $f'_c$ = concrete cylinder strength
- $b$ = width of corbel
- $a$ = depth of concrete compression zone

The horizontal equilibrium of corbel internal force is:

$$\sum H = 0 \Rightarrow C_c = T_b$$

$$0.85f'_c ba(\cos \beta) = A_f f_y$$

$$a = \frac{A_f f_y}{0.85f'_c b(\cos \beta)}$$

The flexure reinforcement area is:

$$A_f = \frac{M_u}{\phi f_y (d - \frac{a}{2})} \quad [11.7]$$
\[ A_f = \frac{M_u}{\phi f_y d} \left( \frac{A_f f_y}{0.85 f_y b \cos \beta} \right) \]

\[ \cos \beta \text{ value can be calculated based on the } \tan \beta \text{ value as follows:} \]

\[ \tan \beta = \frac{jd}{a} \quad [11.8] \]

where:
- \( a \) = distance of \( V_u \) from face of column
- \( jd \) = lever arm

Based on the equation above we must trial and error to find the reinforcement area \( A_f \).

For practical reason the equation below can be used for preliminary:

\[ A_f = \frac{M_u}{\phi f_y (jd)} \quad [11.9] \]

where:
- \( A_f \) = area of flexural reinforcement
- \( M_u \) = ultimate flexure moment at corbel
- \( f_y \) = yield strength of the flexural reinforcement
- \( \phi \) = strength reduction factor (\( \phi = 0.9 \))
- \( d \) = effective depth of corbel

### 11.4.4 Distribution of Corbel Reinforcements

\[ A_s = \frac{2}{3} A_f + A_n \]
\[ A_h = \frac{1}{2} A_f \]

\[ \text{CASE 1} \quad \text{CASE 2} \]

**Figure 11.4 Distribution of Corbel Reinforcements**
From the last calculation we already find the shear friction reinforcement $A_{vf}$, tension reinforcement $A_n$ and flexural reinforcement $A_f$. We must calculate the primary tension reinforcement $A_s$ based on the above reinforcements.

### Table 11.3 Distribution of Corbel Reinforcements

<table>
<thead>
<tr>
<th>CASE</th>
<th>$A_s$</th>
<th>PRIMARY REINFORCEMENT</th>
<th>CLOSED STIRRUP</th>
<th>LOCATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$A_s \geq \frac{2}{3} A_{vf} + A_n$</td>
<td>$A_s = \frac{2}{3} A_{vf} + A_n$</td>
<td>$A_h = \frac{1}{3} A_{vf}$</td>
<td>$\frac{2}{3} d$</td>
</tr>
<tr>
<td>2</td>
<td>$A_s \geq A_f + A_n$</td>
<td>$A_s = A_f + A_n$</td>
<td>$A_h = \frac{1}{2} A_f$</td>
<td>$\frac{2}{3} d$</td>
</tr>
</tbody>
</table>

where:
- $A_s$ = area of primary tension reinforcement
- $A_{vf}$ = area of shear friction reinforcement
- $A_n$ = area of tension reinforcement
- $A_f$ = area of flexure reinforcement
- $A_h$ = horizontal closed stirrup
- $d$ = effective depth of corbel

The reinforcements is taken which is larger, case 1 or case 2, the distribution of the reinforcements is shown in the figure above.

#### 11.4.5 Limits of Reinforcements

The limits of primary steel reinforcement at corbel design is:

$$\rho = \frac{A_s}{bd} \geq \frac{0.04 f'_c}{f_y}$$ \[11.10\]

where:
- $A_s$ = area of primary tension reinforcement
- $b$ = width of corbel
- $d$ = effective depth of corbel

The limits of horizontal closed stirrup reinforcement at corbel design is:

$$A_h \geq 0.5(A_s - A_n)$$ \[11.11\]

where:
- $A_s$ = area of primary tension reinforcement
- $A_n$ = area of tension reinforcement

#### 11.4.6 Step – By – Step Procedure

The followings are the step – by – step procedure used in the flexural design for corbel (bracket), as follows:
Calculate ultimate flexure moment $M_u$:

$$M_u = V_u(a) + N_{uc}(h - d)$$

Calculate the area of tension reinforcement $A_n$:

$$A_n = \frac{N_{uc}}{f_y}$$

Calculate the area of flexural reinforcement $A_f$:

$$A_f = \frac{M_u}{f_y(0.85d)}$$

Calculate the area of primary tension reinforcement $A_s$:

<table>
<thead>
<tr>
<th>CASE</th>
<th>$A_s$</th>
<th>PRIMARY REINFORCEMENT</th>
<th>CLOSED STIRRUP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$A_s \geq \frac{2}{3}A_{df} + A_n$</td>
<td>$A_s = \frac{2}{3}A_{df} + A_n$</td>
<td>$A_h = \frac{1}{3}A_{df}$</td>
</tr>
<tr>
<td>2</td>
<td>$A_s \geq A_f + A_n$</td>
<td>$A_s = A_f + A_n$</td>
<td>$A_h = \frac{1}{2}A_f$</td>
</tr>
</tbody>
</table>

Check the reinforcement for minimum reinforcement:

$$\rho = \frac{A_s}{bd} \geq 0.04 \frac{f_c}{f_y}$$

$$A_h \geq 0.5(A_s - A_n)$$

11.5 APPLICATIONS

11.5.1 APPLICATION 01 – DESIGN OF CORBEL

![Diagram of a corbel with forces and dimensions]
PROBLEM
Design the flexural and shear friction reinforcement of corbel structure above.

MATERIAL
Concrete strength = K – 300
Steel grade = Grade 400
Concrete cylinder strength = $f'_c = 0.83 \times 30 = 24.9 \text{ MPa}$
$\beta_1 = 0.85$

DIMENSION
$b = 200 \text{ mm}$
$h = 400 \text{ mm}$
Concrete cover = 30 mm
$d = 370 \text{ mm}$

DESIGN FORCE
$V_u = 150000 \text{ N}$
$N_{uc} = 0.2V_u = 0.2 \times 150000 = 30000 \text{ N}$
$M_u = V_u(a) + N_{uc}(h - d) = 150000(100) + 30000(400 - 370) = 15900000 \text{ Nmm}$

LIMITATION CHECKING
$\phi(0.2f'_c)b_wd = 0.85(0.2 \times 24.9)200 \times 370 = 313242 \text{ N}$
$\phi(5.5)b_wd = 0.85 \times 5.5 \times 200 \times 370 = 345950 \text{ N}$
$V_u = 150000 < \phi(0.2f'_c)b_wd = 313242 < \phi(5.5)b_wd = 345950$

SHEAR FRICTION REINFORCEMENT
$\mu = 1.4 \lambda = 1.4 \times 1.0 = 1.4$
$A_{vt} = \frac{V_u}{\phi f_y \mu} = \frac{150000}{0.85 \times 400 \times 1.4} = 315 \text{ mm}^2$

TENSION REINFORCEMENT
$A_n = \frac{N_{uc}}{\phi f_y} = \frac{30000}{0.85 \times 400} = 88 \text{ mm}^2$

FLEXURAL REINFORCEMENT
$A_f = \frac{M_u}{\phi f_y(0.85d)} = \frac{15900000}{0.9 \times 400(0.85 \times 370)} = 140 \text{ mm}^2$
**PRIMARY TENSION REINFORCEMENT**

<table>
<thead>
<tr>
<th>CASE</th>
<th>( A_s ) (mm(^2))</th>
<th>PRIMARY REINFORCEMENT (mm(^2))</th>
<th>CLOSED STIRRUP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( A_s \geq \frac{2}{3}A_{nf} + A_n )</td>
<td>( A_s = 298 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( A_s \geq \frac{2}{3}(315) + 88 \geq 298 )</td>
<td>( A_h = \frac{1}{3}A_{nf} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( A_h = \frac{1}{3}(315) = 105 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{2}{3}d )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>247</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( A_s \geq A_{nf} + A_n )</td>
<td>( A_s = 228 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( A_s \geq 140 + 88 \geq 228 )</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>–</td>
<td></td>
</tr>
</tbody>
</table>

The reinforcement of the corbel are:

\( A_s = 298 \) mm\(^2\)

\( A_h = 105 \) mm\(^2\)

**CHECK FOR \( A_s \) MINIMUM AND \( A_s \) MAXIMUM**

\[ \rho_{\text{min}} = 0.04 \frac{f_y}{f_y} = 0.04 \frac{24.9}{400} = 0.00249 \]

\[ \rho = \frac{A_s}{bd} = \frac{298}{200 \times 370} = 0.00402 > \rho_{\text{min}} = 0.00249 \quad \Rightarrow \quad \text{OK} \]

\( A_{h-\text{min}} = 0.5(A_s - A_n) = 0.5(298 - 88) = 210 \) mm\(^2\)

\( A_h = 105 < A_{h-\text{min}} = 210 \Rightarrow A_h = 210 \) mm\(^2\)

The final reinforcement of the corbel are:

\( A_s = 298 \) mm\(^2\)

\( A_h = 210 \) mm\(^2\)

**CORBEL REINFORCEMENT**

<table>
<thead>
<tr>
<th>( A_s ) (mm(^2))</th>
<th>( A_h ) (mm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_s = 3(2 \text{ Legs D10}) )</td>
<td>( A_s = 3D16 )</td>
</tr>
<tr>
<td>( A_s = 3 \left( \frac{1}{4} \pi D^2 \right) = 3 \left( \frac{1}{4} \pi \times 16^2 \right) = 603 )</td>
<td>( A_s = 3 \left( \frac{1}{4} \pi D^2 \right) = 3 \left( \frac{1}{4} \pi \times 10^2 \right) = 471 )</td>
</tr>
</tbody>
</table>
SKETCH OF REINFORCEMENT

2 LEGS Ø10

3D16

247