1- INTRODUCTION

Columns are usually subjected to two bending moments about two perpendicular axes ($X$ and $Y$) as well as an axial force in the vertical $Z$ direction (see Figure 1).

Figure 1: Biaxial bending of columns

(a): 3D view  (b): Bending about $X$-axis
(c): Bending about $Y$-axis  (d): Inclined neutral axis in biaxial bending

With the shown sign convention, bending about $X$-axis causes compression in the top part and tension in the bottom region, whereas bending about $Y$-axis causes compression in the left hand part and tension in the right part. For symmetric sections subjected to uniaxial bending, the neutral axis is parallel to the moment axis. In biaxial bending (d), the top-left part is subjected to double compression and the bottom right part is subjected to double tension. The remaining parts are subjected to combined compression and tension. This means that the two moments are not
independent but coupled. The resulting neutral axis is inclined with an angle depending on the moment values as well as the section properties.

Interaction between the axial force $P$ and the two bending moments $M_x$ and $M_y$ is represented by a 3D surface. The design surface is inside the nominal surface. The 3D surface is constructed by combining several interaction curves $P-M$ at various neutral axis angles.

Various 2D scans can be extracted from the 3D surface:

- Horizontal scan giving interaction curve $M_x - M_y$ for a given value of the axial force $P$, also called load contour.
- Vertical scan giving interaction curve $P - M_x$ for a given value of $M_y$ moment.
- Vertical scan giving interaction curve $P - M_y$ for a given value of $M_x$ moment.
The figure shows a general section subjected to biaxial bending.

With respect to the sign convention shown in the figure, the nominal force and moments in biaxial bending are given by:

\[ P_n = 0.85 f'c B + \sum F_{si} \] \[ M_{nx} = 0.85 f'c B Y_h + \sum F_{si} Y_{si} \] \[ M_{ny} = 0.85 f'c B X_h + \sum F_{si} X_{si} \]

\( B \) is the area of the concrete compression block. \( X_h \) and \( Y_h \) are coordinates of the centroid of the compression block with respect to \( X \) and \( Y \) axes having the origin as the centroid of the gross section. Steel bars are described by their coordinates \( X_{si} \) and \( Y_{si} \).

The compression block may have more than one part as and may contain parts of the possible voids present in the section.

For each neutral axis angle, an interaction curve (meridian) \( P-M_x-M_y \) is constructed by varying the neutral axis depth from pure compression to pure tension. Calculations are complex and are usually carried out in local axes (b).
Biaxial bending analysis and design of columns is very complex and only some specific software can be used for this purpose such as RC-BIAX developed in KSU. Codes of practice such as ACI and SBC allow the use of approximate methods to check for biaxial bending. Among these is the reciprocal method of Bresler.

2- BRESLER RECIPROCAL EQUATION IN BIAXIAL BENDING

\[
\frac{1}{P_n} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_{n0}} \tag{a}
\]

or

\[
\frac{1}{\phi P_n} = \frac{1}{\phi P_{nx}} + \frac{1}{\phi P_{ny}} - \frac{1}{\phi P_{n0}} \tag{b}
\]

\(P_n\): Nominal biaxial strength (unknown)

\(P_{nx}\): Nominal strength with uniaxial bending \(M_x\) only \((M_y = 0)\)

\(P_{ny}\): Nominal strength with uniaxial bending \(M_y\) only \((M_x = 0)\)

\(P_{n0}\): Nominal strength with pure compression \((M_x = M_y = 0)\)

\(\phi\) is the strength reduction factor which should be unique for biaxial bending.

Textbooks use both forms (a) and (b) of Bresler equation, and the second is in fact derived from the first by dividing all terms by the same unique strength reduction factor. Form (b) must be used with caution. In form (b), the design interaction diagrams \(P-M_x\) and \(P-M_y\) are used, and the corresponding strength reduction factors may be different. To avoid confusion, it is therefore recommended to use form (a) directly with the nominal diagrams.

Given the loading \((P_u, M_{ux}, M_{uy})\), we must analyze the column in both directions separately and draw the interaction diagrams \(P-M_x\) and \(P-M_y\). We must check for uniaxial bending that the point \((P_u, M_{ux})\) lies inside the safe region of the \(P-M_x\) diagram and that the point \((P_u, M_{uy})\) lies inside the safe region of the \(P-M_y\) diagram.

For biaxial bending, to use Bresler equation, we must read \(P_{nx}\) on \(P-M_x\) diagram as the nominal force corresponding to the intersection of the nominal diagram with the radial line starting from the origin and passing through the point with coordinates \((M_{ux}, P_u)\). \(P_{ny}\) is read similarly on \(P-M_y\) diagram.

\(P_{n0}\) is either calculated as: \(P_{n0} = 0.85f_c(A_g - A_s) + f_y A_s\) if data is available or simply read on one of the two diagrams as the pure nominal compression strength \((M = 0)\). Bresler equation must then be used to determine the biaxial nominal axial force \(P_n\). We must finally check that \(\phi P_n \geq P_n\).
Example:

Check the safety of the column shown in Figure 2 when subjected to the following loading:

Axial factored compressive load $P_u = 800 \text{kN}$

Factored moment about x-axis $M_{ux} = 120 \text{kN.m}$

Factored moment about y-axis $M_{uy} = 20 \text{kN.m}$

This type of loading, with bending moment about one axis much greater than the moment about the second axis, is frequent in structures.

Check the column in biaxial bending using Bresler method with a strength reduction factor $\phi = 0.65$. The column is 300 x 400 mm with eight 16-mm bars. Concrete cover is 40 mm and the tie diameter is 10 mm. The total area for the eight bars is equal to 1608.50 mm$^2$ representing a ratio of 1.34 % with respect to concrete gross section.

2.1- Bending about X-axis

In this case the section dimensions are: $b = 300 \text{mm}$ and $h = 400 \text{mm}$.

The section has three steel layers:

The layer sections are: $A_{s1} = A_{s3} = 603.186 \text{mm}^2$ and $A_{s2} = 402.124 \text{mm}^2$

The layer depths are: $d_1 = \text{Cover} + \frac{d_b}{2} + d_s = 40 + 8 + 10 = 58 \text{mm}$

$$d_2 = \frac{h}{2} = 200 \text{mm} \quad d_3 = 400 - 58 = 342 \text{mm}$$

The $P$-$M$ interaction diagram about X-axis (produced by RC-TOOL software) is shown in Figure 3.
Figure 3: $P-M_x$ interaction diagram
2.2- Bending about Y-axis

In this case the section dimensions are: \( b = 400\, mm \) and \( h = 300\, mm \).

The section has again three steel layers with the same area values.

The layer depths are:  

\[
\begin{align*}
    d_1 &= 58\, mm \\
    d_2 &= \frac{h}{2} = 150\, mm \\
    d_3 &= 300 - 58 = 242\, mm \\
\end{align*}
\]

The \( P-M \) interaction diagram about Y-axis is shown in Figure 4.

![P-M interaction diagram](image-url)
2.3- Check safety in uniaxial bending
In Figure 3, the point represented by coordinates \(M_{ux} = 120 \text{kN.m} \) and \(P_u = 800 \text{kN}\) lies inside the safe region, although near the border. The column is therefore safe with respect to \(X\)-axis bending.
In Figure 4, the loading point \(M_{uy} = 20 \text{kN.m} \), \(P_u = 800 \text{kN}\) lies inside the safe region. The column is therefore also safe with respect to \(Y\)-axis bending. Both points are in the compression controlled zone corresponding to a strength reduction factor of 0.65. The two dashed radial lines from the origin, shown in Figures 3 and 4 show the balanced point and the 0.005 steel strain point. The two lines represent the boundaries of the transition zone. It can be seen that the first point corresponding to \(M_x\) moment is close to the transition border. This shows that the values of the strength reduction factors in both directions may be different. For this example, with the same moments but an axial force of 300 \text{kN}, the \(M_x\) point would be in the tension controlled zone while the \(M_y\) point would still remain in the compression controlled zone.

2.4- Check biaxial bending using Bresler equation
a/ Determine \(P_{nx}, P_{ny}\) and \(P_{n0}\) using the given diagrams.
Locate the point corresponding to the known ultimate axial force and bending moment and then draw a radial line from the origin passing through this point. The nominal values \(P_{nx}\) and \(P_{ny}\) are given by the intersections of these radial lines with the nominal diagrams.
We read in Figures 5 and 6 that \(P_{nx} = 1262.3 \text{kN}\) and \(P_{ny} = 2587.0 \text{kN}\)
RC-TOOL software may also be used to determine these values with more accuracy.
The RC-TOOL values are: \(P_{nx} = 1262.31 \text{kN}\) \(P_{ny} = 2587.01 \text{kN}\)
The nominal compression force may be estimated by \(P_{n0} = 0.85f_y(A_g - A_{st}) + f_yA_{st}\) or simply be read from the diagrams. We find that: \(P_{n0} = 3191.39 \text{kN}\).
Remark: If the column reinforcement was unknown, it could be determined from this nominal compression force \(P_{n0}\) or from the tensile nominal strength as: \(P_{nt} = -f_yA_{st}\) thus \(A_{st} = -\frac{P_{nt}}{f_y}\)
b/ Determine \(P_n\) using Bresler reciprocal equation
\[
\frac{1}{P_n} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_{n0}} = \frac{1}{1262.31} + \frac{1}{2587.01} - \frac{1}{3191.39} = 0.000865401972
\]
Thus \(P_n = 1155.53 \text{kN}\)
Figure 5: Determination of $P_{nx}$ from $P-M_x$ interaction diagram
c/ Compare design $\phi P_n$ and ultimate $P_u$ values

$\phi P_n = 0.65 \times 1155.53 = 751.1 \, kN$

This value is less than the ultimate axial force (800 kN).

$\phi P_n < P_u$ The column is therefore unsafe and must be redesigned.

(It is biaxially unsafe although it is uniaxially safe in both X and Y directions).

This example shows how important biaxial bending is.

The approximate methods such as Bresler equation are not valid for all loading cases and types of sections. Bresler method is usually conservative and convenient for circular, square and near-square sections. The value of the strength reduction factor to use is not always obvious, especially if the two bending planes give different values.

Use of software is therefore encouraged for analysis and design of columns subjected to biaxial bending combined with an axial force.
3- USE OF RC-BIAX SOFTWARE

3.1- Presentation of RC-BIAX software

This software was developed at KSU by Professor Abdelhamid Charif and performs both analysis and design of various types of sections subjected to biaxial bending combined with an axial force. Any section shape may be considered. The steel reinforcement must be described by bars and not layers. Bar coordinates are then required. Many bar generation options are available.

Figure 7: Biaxial bending analysis of the previous example
3.2 - Section Analysis and Check

The previous example was analyzed using RC-BIAx software with the same data. Figure 7 shows the section data as well as the nominal and design interaction surfaces. The user can check any loading combination and extract various 2D scans from the 3D surface. Figure 8 shows a 2D scan for an axial force equal to the ultimate value used previously \( P_u = 800 \text{ kN} \) and checks safety for the loading combination. The scan is shown in perspective and as a 2D \( M_x-M_y \) interaction curve. The loading point lies outside the safe domain, thus confirming that it is unsafe. The software delivers the ray ratio between the strength capacity and the loading point (a safe combination corresponds to a ratio greater or equal to unity).
RC-BIAX software also gives the results of approximate methods such as Bresler reciprocal equation and the equivalent eccentricity method. It allows therefore the user to investigate the limitations of these approximate methods. The previous example results can be retrieved by the software. For the strength reduction factor (which can be different in both planes as seen earlier) to be used in Bresler equation, RC-BIAX considers the average value.

It can be shown using RC-BIAX that, for instance, that the loading combination:

\[ P_u = 1500 \text{ kN} \quad M_{ux} = 50 \text{ kN.m} \quad M_{uy} = 40 \text{ kN.m} \]

is safe but that Bresler method delivers an unsafe check. This proves that this method is not always conservative even for rectangular sections.

### 3.3- RC Design using RC-BIAX software

The software can also be used for design. The steel pattern must then be entered. Using the same previous reinforcement pattern (eight similar bars with the same ratio), it was found that required steel was about 1.76 % of the gross section, corresponding to eight 20-mm bars (actual ratio of 2.09 %). Use of eight 18-mm bars (1.70 %) is not sufficient and the section would remain unsafe in biaxial bending. Figure 10 shows design results for a different steel pattern in order to allow for the great value of bending moment \( M_x \). Ten bars are used with four similar bars at the top and bottom. The two side bars have a smaller ratio than the top and bottom bars. The required steel ratio (1.58 %) is less than the previous one. Eight 16-mm bars (four at top and four at bottom) and two 14-mm side bars would be then sufficient. This shows that RC-BIAX software can be used not only to design sections under biaxial bending but also to optimize steel distribution through steel pattern variation. The software also delivers very useful information about the neutral axis depth and angle as well as strains and stresses in concrete and all steel bars. Figures 8 and 9 show other design examples including complex sections with holes. For these sections no approximate method is available. Figure 8 shows that in an unsymmetrical section (L section) subjected to a single moment \( M_x \) only \( (M_y = 0) \), the neutral axis is inclined and not parallel to the moment axis as in symmetric sections. The resultant of the bar forces and the concrete compression force lies on the vertical centroid Y-axis \((e_x = 0)\) corresponding to zero moment about Y-axis.

![Figure 8: Design of an L section](image)
Figure 9: Design of a complex section with holes in biaxial bending

Neutral axis angle = 57.1 deg
Rotated section
+ + Mx1 = My1
Loading angle = 45.0 deg
Modified loading angle = 52.9 deg

Figure 10: Design of the section for the same loading but a different reinforcement pattern

Pu (kN) = 800.000
Mx1 (kN.m) = 120.000
My1 (kN.m) = 20.000
Conc. corr. stress (MPa) = 21.2300
Compression-controlled section
Neutral axis angle (deg) = 24.4982
Neutral axis depth (mm) = 267.0026

Top & bottom bars 1 to 8:
Area = 199.611 mm²
Side bars 9 and 10:
Area = 152.827 mm²

STEEL 1 Depth (mm) = 336.2757
Strain = -0.00077
Stress (MPa) = -153.1956
STEEL 2 Depth (mm) = 360.8507
Strain = -0.002105
Stress (MPa) = -210.3105
STEEL 3 Depth (mm) = 366.1237
Strain = -0.00134
Stress (MPa) = -267.4254
STEEL 4 Depth (mm) = 411.5477
Strain = -0.00162
Stress (MPa) = -324.5403
STEEL 5 Depth (mm) = 76.8245
Strain = 0.00214
Stress (MPa) = 420.0000

Concrete f'c (MPa) = 25.00
Beta = 0.8530
Steel fy (MPa) = 420.00
Gross sections properties:
Area (mm²) = 120000.00
Centroid coordinates:
Xc (mm) = 160.000
Yc (mm) = 200.000
Centroid moments of inertia:
Ix (mm⁴) = 0.16000E+10
Ly (mm⁴) = 0.00000E+00
Total steel (mm²) = 1690.952
Steel percentage = 15.025 %