Simplified semi-analytical model for elastic distortional buckling prediction of cold-formed steel flexural members

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ABSTRACT

The expressions for elastic distortional buckling stress predictions available in literature are used with reasonable accuracy, but are either iterative or involve increased calculation effort for design. It is an accepted fact among researchers that these expressions are not evaluated enough for the distortional stress prediction of sections with complex lip stiffeners. In this paper, a candidate model for distortional buckling stress predictions is presented which is semi analytical in nature and is simple to incorporate in the direct strength method (DSM) of cold-formed steel design. The proposed expression incorporates the effect of complex lip stiffeners on the elastic distortional buckling capacities of cold formed steel flexural members. In the proposed model, the (i) translational stiffness at lip-flange junction and (ii) rotational stiffness at the flange-web junction, are derived from regression analysis of wide range of cross sectional dimensions. The proposed method is calibrated with semi-analytical finite strip method presented in the literature and this formulation has been demonstrated to be good in comparison with recently published numerical based distortional buckling predictions.

1. Introduction

The migration in design philosophy of cold-formed steel sections from traditional effective width method to direct strength method necessitates the evaluation of elastic buckling stresses such as (i) local buckling, (ii) distortional buckling and (iii) overall Euler buckling stress using numerical methods like finite element method, finite strip method or by using analytical expressions. Analytical expressions are available in Timoshenko and Gere [1] for the evaluation of elastic local and Euler (global) buckling stresses whereas distortional buckling predictions are usually based on analytical model of flange-lip combination with rotational and translational stiffness at junction point. Distortional buckling phenomenon is characterized by the rotation of the flange-lip combination about the flange-web junction and the analytical model is designed to incorporate this mechanism.

There are several papers on the prediction of distortional buckling stresses which is applicable for the design procedures presented in codes of practice. One of the earliest work related to distortional buckling was on steel storage rack columns by Hancock [2], providing design chart for buckling coefficient and half buckling wavelength for a range of section geometries. Kwon and Hancock [3] provided design equations for distortional buckling

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buckling prediction, Lau and Hancock [11] derived analytical expression for elastic distortional buckling stress of compression members using flexural torsional buckling theory developed by Timoshenko and Gere [1] and Vlasov [12]. The derivations were based on approximate model of flange-lip combination rotating about flange-web junction. The theory has been extended by Hancock [13] to flexural members wherein a design method for flexural members was provided. Iterative calculations are involved in the determination of rotational stiffness terms provided in [11,13]. Analytical expression for sections with laterally unsupported compression flanges for evaluating elastic distortional buckling stress based on the assumption of beam on elastic foundation was presented by Serrette and Pekoz [14]. A closed form solution for distortional buckling based on a model of flange-lip system, with tension end of web treated as pinned was presented by Davies and Jiang [15]. Rogers and Schuster [16] compared various analytical models on distortional buckling of flexural members and established an improved prediction by Hancock model on elastic distortional buckling strength. The analytical model proposed by Schafer and Pekoz [17] considered the interaction model of web and flange by incorporating the elastic stiffness and geometric stiffness portion of both flange and web for distortional buckling strength calculation. The web stiffness contribution for the formulation were derived using numerically based semi-analytical finite strip method. Teng et al. [18] provided analytical expression for evaluating elastic distortional buckling strength of sections subjected to axial compression and biaxial bending by extending Lau and Hancock [11] method. Silvestre and Camotim [19] provided analytical formula for distortional critical length and buckling stress for cold-formed steel channel and zed section members under uniform compression, pure bending and both using generalized beam theory. A model for calculating distortional buckling stress of channel, zed and sigma section under compression or bending about an axis perpendicular to the web was proposed by Li and Chen [20]. The analytical model consists of lip-flange combination incorporating a translational spring instead of rotational spring as adopted in Lau and Hancock [11] model. Tong et al. [21] proposed a unified analytical model for the distortional and lateral buckling analysis of channel beams. Salient points of development in the analytical models for distortional buckling stress prediction are briefly compared in the next section.

2. Critical evaluation of existing formulations on elastic distortional buckling stress

The mechanics involved in the distortional buckling of cold-formed steel flexural member is shown in Fig. 1. In an analytical model, the deformation of cross section as a result of distortional buckling is idealized as flange-lip combination under uniform compression with rotational (\(k_{\phi}\)) and translational (\(k_x\)) stiffness. The equations of equilibrium by considering forces along x and y direction and moment about shear centre are formulated using flexural torsional buckling theory. The partial differential equations are solved to determine the elastic distortional buckling load. The role of \(k_{\phi}\) is to incorporate the rotational restraint offered by web on web-flange junction and the resistance offered by web is also dependent on the fixity at tension flange and stress gradient in the web. The effect of \(k_x\) has been neglected in majority of reported research work [11,13,15,20] and its effect on elastic distortional buckling strength of flexural members has been neglected in this paper also. The existing formulations are based on flanges under uniform compression and hence the capability of these formulations to capture the effect of stress gradient in the flange on distortional buckling is assessed here in this study.

A comparative study of existing analytical models on elastic distortional buckling prediction of flexural members has been performed. Two analytical models for distortional buckling of flexural members; Hancock [13] and Davies and Jiang [15] (which will hitherto be called as Hancock model and Davies model, in the present study) are compared for the evaluation of elastic distortional buckling stress under flexure. The geometry of lipped channel cross sections for the comparison of analytical models are taken from the work of Hancock [13] and Schafer and Pekoz [17] and the results are compared with finite strip analysis (FSM) results using the software CUFSM developed by John Hopkins university [22]. The comparison of distortional buckling stress predictions of analytical models with finite strip results with variation in flange width to lip depth ratio (b/c) in a non-dimensional form and the statistical comparison of results is shown in Fig. 2. It should be noted that the range of sections that are considered for the comparison has compactness ratio (b/h) varying from 0.13 to 0.75. The flange width to lip depth ratio (b/c) considered is between 2 and 16. In direct strength method, a value of b/c > 1.4 is accepted as within the pre-qualified range.

Elastic distortional buckling calculation using analytical models are comparable with finite strip results over a range of cross section dimensions and relatively better estimate of results are available for b/c value greater than 5. For b/c value less than 5, the analytical predictions were mostly lower bound to finite strip results and the percentage variation is nearly 40%. For a few section geometries, Hancock and Davies models produces negative rotational stiffness values at web-flange junction resulting in very high elastic distortional buckling strength prediction. In the expression presented by Hancock model and Davies Model in literature, one could see that the multiplier bracketed term for the rotational stiffness \(k_{\phi}\), represents the reduction in the flexural stiffness offered by the web due to axial stresses. However when this term becomes negative, the reduction is completely neglected. This in turn results in prediction of higher capacities, which is evident from Fig. 2.

Comparative study has been done on lipped channel cross section to assess the effect of section geometries on distortional buckling stress with compactness ratio (b/h), by varying the width (b) of the section keeping other cross sectional dimensions constant (h = 300 mm, c = 50 mm and t = 1 mm). The values calculated using theoretical models are compared with distortional buckling stress calculated using finite strip analysis as shown in

![Fig. 1. Distortional mechanics of cold-formed steel beams under flexure [11].](image)
Fig. 3. The distortion by Hancock model are lower bound to finite strip results for the considered sections and as the ratio of width of flange to depth of web (b/h) increases, the variation in results between analytical predictions from finite strip method decreases. The distortional buckling strength prediction by Davies model follows the same trend as Hancock model and the variation of analytical values from finite strip analysis results for the model are less than 10% for b/h ratio greater than 0.5.

The effect of stress gradient in flanges on elastic distortional buckling stress has been evaluated by comparing finite strip analysis on sections subjected to major and minor axis bending. The stress distribution in the cross section (compression denoted by ‘−’ sign and tension by ‘+’ sign) subjected to major axis and minor axis bending and the comparison of finite strip results with theoretical predictions for various half buckling wave length for lippered channel section is shown in Fig. 4. Even though the elastic distortional buckling curve in theoretical predictions and finite strip method are differing, not much variation has been observed for sections subjected to major axis and minor axis bending for the section compared.

The comparison of distortional buckling curve between analytical predictions and finite strip analysis for a variety of cross sections under flexure has been done and the result corresponding to a rack section with complex stiffener is shown in Fig. 5. The distortional buckling curve predicted by analytical models and finite strip results are not identical for the section compared and variation in critical buckling stress is also observed. This points to the fact that the analytical predictions published in the literature do not adequately capture the effect of complex lip stiffeners in elastic distortional buckling stress. Hence the present study makes an attempt to incorporate the effect of complex lip stiffeners on elastic distortional buckling formulation.

3. Semi analytical model for elastic distortional buckling stress for beams

A semi analytical model of flange-lip combination (Fig. 6) has been proposed incorporating the effect of lip stiffeners on distortional buckling in the form of translational stiffness in addition to
translational and rotational stiffness available in the existing analytical models. The additional translational stiffness \((k_x)\) is attached to the flange-lip junction to take into account the translational restraint offered by the compound lips.

The axial buckling load, \(P\) is determined by three simultaneous differential equations by considering equilibrium of forces along \(x\) and \(y\) direction and equilibrium of moment about shear centre axis.

\[
E_I \frac{d^4 u}{dz^4} + E_l \frac{d^4 v}{dz^4} + P \left[ \frac{d^2 u}{dz^2} + y_0 \frac{d^2 \phi}{dz^2} \right] + k_u \left[ u + (y_0 - h_y) \phi \right] = 0
\]  
(1)

\[
E_I \frac{d^4 v}{dz^4} + E_l \frac{d^4 u}{dz^4} + P \left[ \frac{d^2 v}{dz^2} - x_0 \frac{d^2 \phi}{dz^2} \right] + Q_y + k_v \left[ v + (b - x_0 + h_x) \phi \right] = 0
\]  
(2)

\[
E_I \frac{d^4 \phi}{dz^4} - \left( GJ - \frac{h_y}{A} \right) \frac{d^2 \phi}{dz^2} - \left[ \pi^2 \frac{E_l I_y}{x_0} - \pi^2 \frac{E_l I_y}{h_y} \right] - Q_y (x_0 - h_y)
+ k_\phi \left[ u + (y_0 - h_y) \phi \right] (y_0 - h_y)
+ k_\phi \left[ v + (b - x_0 + h_x) \phi \right] (b - x_0 + h_x) + k_\phi \phi = 0
\]  
(3)

The three differential equations are simplified to obtain the quadratic equation for distortional buckling stress. Let

\[
\phi = A_1 \sin \left( \frac{sz}{\lambda} \right)
\]  
(4)

\[
u = A_2 \sin \left( \frac{sz}{\lambda} \right)
\]  
(5)

The effect of translational stiffness \(k_x\) on distortional buckling appears to be small and hence neglected in simplified formulation. Also the \(y\)-direction displacement along elastic support is zero. Therefore

\[
v = (x_0 - h_x) A_1 \sin \left( \frac{sz}{\lambda} \right)
\]  
(6)

Substituting Eqs. (4), (5) and (6) in (1), (2) and (3) and making suitable simplifications

\[
\phi \left[ \pi^2 \left( x_0 - h_x \right) E_l I_y - P y_0 \right] + u \left[ \pi^2 \frac{E_l I_y}{x_0} - P \right] = 0
\]  
(7)

\[
\phi \left[ \pi^2 \left( x_0 - h_x \right) E_l I_y - P \left( x_0 - h_x \right) + 2 P y_0 (x_0 - h_y) + E_l \frac{\pi^2}{x_0} + G J
- P \frac{h_y}{A} \frac{J^2}{\pi^2} \left( k_y b^2 + k_y \right) \right] + u \left[ \frac{\pi^2}{x_0} (x_0 - h_x) E_l I_y - P y_0 \right] = 0
\]  
(8)

From Eqs. (7) and (8)

\[
\pi^2 \left( x_0 - h_x \right) E_l I_y - P y_0 \left[ \frac{\pi^2}{x_0} \frac{E_l I_y}{x_0} - P \right] - \pi^2 \frac{E_l I_y}{x_0} + E_l \left( x_0 - h_x \right) \right] + G J - P \left[ \frac{h_y}{A} - x_0 + h_x \right] + \frac{J^2}{\pi^2} \left[ k_y b^2 + k_y \right] \right] = 0
\]  
(9)

Simplifying and expressing in the form of quadratic equation

\[
P^2 \left[ y_0^2 - \frac{h_y}{A} + x_0^2 - h_x^2 \right] + P \left[ \frac{\pi^2}{x_0} E_l - 2 y_0 \frac{\pi^2}{x_0} \right] E_l I_y (x_0 - h_x) + \frac{\pi^2}{x_0} E_l (x_0 - h_x)^2 + G J
+ \frac{J^2}{\pi^2} \left( k_y b^2 + k_y \right) + \frac{\pi^2}{x_0} E_l \frac{h_y}{A} - \frac{\pi^2}{x_0} E_l \left( x_0 - h_x \right)^2 + G J
+ \frac{\pi^2}{x_0} E_l I_y (x_0 - h_x)^2 - \frac{\pi^2}{x_0} E_l \left( x_0 - h_x \right)^2 - \frac{\pi^2}{x_0} E_l I_y (x_0 - h_x)^2
- \frac{\pi^2}{x_0} E_l G J - E_l \left( k_y b^2 + k_y \right) = 0
\]  
(10)

The quadratic equation has to be solved to determine the critical elastic distortional buckling load, \(P\). The critical half buckling wave length \(\lambda_{cr}\) is calculated by the procedure discussed below.

### 3.1. Calculation of critical half buckling wave length

For the calculation of critical half buckling wave length, assuming the translational stiffness \((k_x)\) as infinity. Then we have

\[
\phi = A_1 \sin \left( \frac{sz}{\lambda} \right)
\]  
(11)

\[
u = (x_0 - h_x) \phi
\]  
(12)

\[
\nu = (x_0 - h_x) \phi
\]  
(13)

Substituting Eqs. (11), (12) and (13) in (1), (2) and (3) and simplifying the equations

\[
P \left[ -h_y \left( y_0 - h_y \right) \right] = \frac{\pi^2}{x_0} \left[ \frac{l_y (y_0 - h_y)^2 - l_y y_0 (x_0 - h_x)(y_0 - h_y)}{1} \right]
\]  
(14)

\[
\frac{\pi^2}{x_0} \left[ l_y (x_0 - h_x)^2 - l_y y_0 (x_0 - h_x)(y_0 - h_y) + l_y + \frac{J^2}{\pi^2} \left( k_y b^2 + k_y \right) \right]
\]  
(15)

From Eqs. (14) and (15)

\[
\frac{\pi^2}{x_0} E_l I_y + G J + \frac{J^2}{\pi^2} \left( k_y b^2 + k_y \right) = \frac{l_y + \frac{J^2}{\pi^2} \left( k_y b^2 + k_y \right)}{x_0 + h_y^2 + h_x^2}
\]  
(16)

where

\[
l_y = l_y + l_y (x_0 - h_x)^2 + l_y \left( y_0 - h_y \right)^2 - 2 l_y y_0 (x_0 - h_x)(y_0 - h_y)
\]  
(17)

Therefore

\[
P = \frac{\pi^2}{x_0} E_l I_y + G J + \frac{J^2}{\pi^2} \left( k_y b^2 + k_y \right)
\]  
(18)

For the critical buckling load to be minimum, taking derivative of \(P\) with respect to half buckling wave length; \(\frac{\pi^2}{x_0} = 0\) and simplifying

\[
\lambda_{cr} = \frac{\pi^2}{k_y b^2 + k_y}
\]  
(19)

The expression for \(k_y\) and \(k_x\) has been determined based on plate theory with appropriate reduction factors which is explained in the next section.
Table 1
Statistical comparison of proposed model with finite strip method.

<table>
<thead>
<tr>
<th></th>
<th>( \sigma_{\text{Schafer}} / \sigma_{\text{fsm}} )</th>
<th>( \sigma_{\text{proposed}} / \sigma_{\text{fsm}} )</th>
<th>( \sigma_{\text{Schafer}} / \sigma_{\text{fsm}} )</th>
<th>( \sigma_{\text{proposed}} / \sigma_{\text{fsm}} )</th>
<th>( \sigma_{\text{Schafer}} / \sigma_{\text{fsm}} )</th>
<th>( \sigma_{\text{proposed}} / \sigma_{\text{fsm}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.88</td>
<td>0.92</td>
<td>0.78</td>
<td>0.85</td>
<td>0.94</td>
<td>0.98</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.09</td>
<td>0.16</td>
<td>0.12</td>
<td>0.15</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>( \text{cov} )</td>
<td>0.10</td>
<td>0.17</td>
<td>0.15</td>
<td>0.18</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>( \text{min} )</td>
<td>0.68</td>
<td>0.57</td>
<td>0.51</td>
<td>0.58</td>
<td>0.69</td>
<td>0.53</td>
</tr>
<tr>
<td>( \text{max} )</td>
<td>1.08</td>
<td>1.20</td>
<td>1.01</td>
<td>1.17</td>
<td>1.32</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Fig. 7. Experimental sections for comparison (Kesti and Davies [5]).

Fig. 8. Sections with complex edge stiffeners (Schafer et al. [25]).
3.2. Calculation of \( k_p \) and \( k_y \)

The rotational stiffness \( k_p \) and translational stiffness \( k_y \) depends mainly on the fixity at flange-web junction on both tension and compression end and the amount of complexity of lips respectively. The actual rotational restraint offered by flange-web junction to distortional buckling depends on the \( b/h \) ratio, slenderness of flange and web. To calibrate \( k_p \) and \( k_y \), a range of lipped channel, rack and hat sections within the bounds of direct strength method has been analysed using proposed formulation and compared with finite strip results. A statistical comparison of proposed formulation and finite strip results are made in Table 1 to arrive at an empirical expression for \( k_p \) and \( k_y \) as shown in Eqs. (20)–(23). The range of sections that are considered for the comparison has compactness ratio \( (b/h) \) varying from 0.3 to 0.7. This range is reflected in the cross sections used in practice as given in AISI design manual [23]. The flange width to lip depth ratio \( (b/c) \) considered is between 1.5 and 10.

\[
\text{For } 4 \left( \frac{b}{h} \right)^{1.6} < 1 \quad k_p = \frac{16D}{h} \left( \frac{b}{h} \right)^{1.6} \tag{20}
\]

\[
\text{For } 4 \left( \frac{b}{h} \right)^{1.6} \geq 1 \quad k_p = \frac{4D}{h} \tag{21}
\]

\[
\text{For } 4 \left( \frac{c}{h} \right)^{5} < 1 \quad k_y = \frac{4D}{h^{3}} \left( \frac{c}{h} \right)^{5} \tag{22}
\]

\[
\text{For } 4 \left( \frac{c}{h} \right)^{5} \geq 1 \quad k_y = \frac{D}{h^{3}} \tag{23}
\]

here \( c \) is the total length of the edge stiffener

The proposed model has been compared with the model developed by Schafer and Pekoz [17] (Schafer model) which is partially based on semi-analytical finite strip method. The calculations involved in the determination of web elastic and geometric stiffness terms in Schafer model appears to be lengthy and complex in nature. From the overall comparison of finite strip results with analytical predictions, it has been demonstrated that the elastic distortional buckling prediction by the proposed model provides a close comparison with finite strip method. Another advantage is the non-iterative nature of the proposed model in the calculation of rotational stiffness term \( (k_y) \). The proposed model provides un-conservative predictions for certain geometries when the \( b/c \) ratio is greater than 5, where the stiffness contribution by the lip on to the flange is negligible.

3.3. Simplified expression for elastic distortional buckling

The quadratic expression in Eq. (10) is solved and simplified for application in design using DSM. The expressions are non-iterative in nature and a new terminology \( K \) (total restraint) is also introduced. A simplified expression for the calculation of elastic distortional buckling load \( (P_{cr}) \) for cold-formed steel beams is shown below. The Eq. (24) has been found to be robust as the terms under square root is always positive for the range of geometries considered.

\[
P_{cr} = \frac{\alpha_3 \pm \sqrt{\alpha_3^2 - 4\alpha_4 \alpha_5}}{2\alpha_4}
\tag{24}
\]

\[
\eta = \left( \frac{\pi}{\alpha_{cr}} \right)^2
\tag{25}
\]

\[
K = k_y b^2 + k_p
\tag{26}
\]

\[
\alpha_4 = \frac{\left( I_y + I_x \right)}{A} - h_x^2
\tag{27}
\]

\[
a_2 = l_w + l_4(x_0 - h_x)^2 + GJ \eta + k \frac{K}{\eta 
\tag{28}
\]

\[
\alpha_3 = \eta F \left[ 2y_0 l_y \left( x_0 - h_x \right) - a_2 + l_4 \left( \alpha_3 - y_0^2 \right) \right]
\tag{29}
\]

\[
a_4 = \eta F \left[ l_y^2 \left( x_0 - h_x \right)^2 - l_y \alpha_2 \right]
\tag{30}
\]

The values of \( k_p \) and \( k_y \) are the proposed translational spring stiffness in ‘y’ direction at the flange-lip junction and rotational stiffness at the web-flange junction respectively. These are calculated as per Eqs. (20)–(23).

4. Application of proposed formulation in the calculation of distortional buckling stress

To assess the applicability of proposed formulation for sections available in cold-formed steel industry and literature, a variety of sections are used for the calculation of elastic distortional buckling stress of flexural members using the proposed model and comparison with finite strip results. The industrial sections used for comparison has compactness ratio \( (b/h) \) between 0.3 and 0.7 and flange width to web depth ratio \( (b/c) \) ranges between 3.1 and 3.3. The experimental sections by Kesti and Davies [5] shown in Fig. 7 has been analysed for \( b/h \) within the range 0.3 and 0.7, whereas the \( b/c \) values are between 1.6 and 6.2. The study by Schafer et al. [25] on cross sections including complex lip stiffeners shown in Fig. 8 are compared for \( b/h \) values between 0.3 and 0.7, whereas the \( b/c \) values are within the range 2 and 5.3.

The comparison of proposed formulation with finite strip results for sections with simple lip stiffeners is shown in Table 2. The prediction of distortional buckling stress by proposed formulation is comparable with finite strip results for all the sections demonstrated in Table 2. The applicability of the proposed model for section with inclined stiffeners and stiffened web is also verified here. For experimental hat and CH2 sections [as defined by Kesti and Davies [5]], and also for sections in [25], the prediction by proposed formulation is upper bound for certain geometries which is observable when the \( b/c \) ratio is greater than 5.

To have a deeper assessment of the distortional buckling strength prediction of proposed formulation for sections involving complex lip stiffeners, a range of beam sections having complex lip geometries are compared with finite strip results and is shown in Table 3. The proposed model consistently predicts the distortional buckling stress closer to finite strip results for all the sections compared. Even though the rack sections shown in [5] are predominantly for compression members, the distortional buckling expression for flexure may be applicable for the design of beam-columns of those sections using direct strength method. From the comparison of sections in [25], it has been observed that the accuracy of prediction by proposed model increases as the complexity of the lip stiffeners increases.
5. Range of validity of the proposed expression

A comparison of proposed formulation with finite strip results for various cross section geometries having b/h value within the range 0.3 and 0.7 is shown in Fig. 9. Within the range shown in Fig. 9, the predictions made by the proposed formulation is mostly conservative and matching reasonably with finite strip analysis results. Even though the coefficient of variation of the proposed model is marginally higher than Schafer model (same in the case of Tables 1, 2 and 3), the mean values (first order variable) are the most important parameter to be considered in statistical analysis which matches well with finite strip results. The bias in the present model is 0.05 whereas in Schafer model it is 0.13. It is shown in Fig. 9 that the present model represent the mean of large number of results in the band. Hence the proposed model can be considered as yet another candidate model for the elastic distortional buckling stress prediction. The proposed formulation is directly applicable for the evaluation of distortional buckling stress of flexural members in direct strength method of design within this range.

6. Conclusions

A review of the existing theoretical models for calculating elastic distortional buckling stress of cold formed steel beams have been made and compared with finite strip analysis results. The inadequacies in the published theoretical models in calculating distortional buckling stress has been assessed. A semi-analytical model of distortional buckling for flexural member is proposed incorporating the effect of lips on elastic distortional buckling stress. The model involves non iterative calculation and is found to be satisfactory in predicting the distortional buckling stress of sections having b/h ratio within the range 0.3 and 0.7 compared to other theoretical models. The proposed formulation is able to provide closer comparison with finite strip results for variety of sections including rack sections and sections with complex lip

Table 2
Comparison of proposed formulation for sections with simple lips.

<table>
<thead>
<tr>
<th>Industrial sections [24] (channel and zed sections)</th>
<th>Kesti and Davies [5] (C, HAT and CH2 sections)</th>
<th>Schafer et al. [25] (section a &amp; b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>μ</td>
<td>0.89</td>
<td>0.91</td>
</tr>
<tr>
<td>σ</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>cov</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>min</td>
<td>0.88</td>
<td>0.83</td>
</tr>
<tr>
<td>max</td>
<td>0.90</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table 3
Comparison of proposed formulation for sections with complex stiffeners.

<table>
<thead>
<tr>
<th>Kesti and Davies [5] (Experimental RA and RL sections)</th>
<th>Schafer et al. [25]</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>14</td>
</tr>
<tr>
<td>μ</td>
<td>0.71</td>
</tr>
<tr>
<td>σ</td>
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</tr>
<tr>
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<td>0.64</td>
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<td>max</td>
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Fig. 9. Statistical comparison of models within the valid range (a) Schafer model, (b) Proposed model.
stiffeners. The formulation has been demonstrated to be a simple and robust tool in predicting elastic distortional buckling of flexural members having generic cross sections.

References